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# Multi-Objective Routing in Integrated Services Networks: A Game Theory Approach

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**Abstract**

In this paper, we consider the multi-objective routing problem in multiple class Integrated Services Networks. We introduce a multi-server two class queueing model where packets from the first class can be queued, while packets from the other class are blocked when the number of packets in the system exceeds some threshold. Therefore, the first class wants to minimize its average packet delay, while the other class wants to minimize its blocking probability.

We formulate the resulting multi-objective routing problem as a Nash game, where each class tries to minimize its own cost function in competition with the other class. We derive the routing policy for a two server parallel system and show the strategy and performance of each class.

## 1 Introduction

Traditionally, there were separate networks (circuit/packet switched) for carrying different traffic types (voice/data). In design and control problems of such networks, the goal is optimization of a single performance objective for the traffic type carried by the network under consideration. For example, in circuit switched networks, a voice call should be guaranteed an acceptable delay or else it should be blocked. The objective is then to minimize the voice blocking probability. In packet switched networks, a data packet may be queued in buffers at intermediate nodes, thus the objective is to minimize the packet delay.

Current trends are for a single high speed packet switched network, called Broadband Integrated Services Digital Network (B-ISDN), that will support multimedia traffic (voice, data, video, etc.) simultaneously. These multiple classes of traffic will share the same network resources (buffers, switches, transmission lines, etc.) for flexible and efficient resource sharing. However, each class has different and conflicting performance requirements and objectives to those of other classes. Hence new methodologies are needed for network design and control problems.

In this paper, we present an approach for the multiple class routing problem based on game-theory and we explicitly solve the routing problem for two classes of packets that share two links, (see Figure 1). One class of packets may be queued at the link buffers, while the second class of packets are blocked when there is not enough space. The objective for the first class is to minimize the delay for its packets, while the objective for the second class is to minimize its blocking probability. An application of this problem is to data/voice packet switched networks, where data packets may be queued at the links, while voice packets are dropped when they estimate that they will experience unacceptable delay. Another application is to ATM networks, where regular packets may be queued at the link buffers, while marked packets are dropped when there is congestion. Another application is for networks shared by different vendors, where the first vendor has unlimited access to the links, while the second vendor may use the links only if the congestion level is below a threshold.

The routing problem has been formulated as a Nash game by Economides and Silvester [9, 8, 10] and Bovopoulos [2, 3]. Another problem that has been recently formulated as a Nash game is the flow control [5, 3, 4, 6, 13, 12, 11, 14] and the joint load sharing, routing and congestion control problem [10].

In section 2, we introduce a multi-server two-class queueing model, where packets from the first class are queueable, while packets from the other class are blocked. In section 3, we formulate the routing problem through a parallel system of such multiserver queueing systems as a Nash game. In section 4, we explicitly solve a routing problem for two queues. Finally in section 5, we conclude on the proposed alternative formulation for the routing problem.

## 2 Multi-Server Queues with Blocking

We consider a queueing system of $m$ servers that are shared by packets from two classes $\alpha$ and $\beta$. Class $\alpha$ packets arrive at rate $\lambda^\alpha$ (Poisson) and if all servers are busy, then they queue in the single queue of the system. Class $\beta$ packets arrive at rate $\lambda^\beta$ (Poisson) and if there are more than $K$ packets (in queue and in service), then they are rejected. We select units so that the length of each packet is exponentially distributed with mean 1 and the rate of each server is $1/\mu$. A packet receives service from a server chosen randomly among the free servers. Furthermore, for stability reasons it is assumed that the total arrival rate is less than the total service rate : $\lambda^\alpha + \lambda^\beta P[n < K] \leq m\mu$.

So, class $\alpha$ packets can be queued, while class $\beta$ packets are blocked. Therefore, a reasonable objective is for class $\alpha$ to minimize its average packet delay, and for class $\beta$ to minimize its blocking probability.

For the above queueing system, we consider two cases:

i) $K \geq m$, i.e. the blocking threshold for class $\beta$ packets is greater than or equal to the number of servers.

ii) $K \leq m$, i.e. the blocking threshold for class $\beta$ packets is less than or equal to the number of servers.

### 2.1 $K \geq m$

Let first consider the case where the blocking threshold $K$ for class $\beta$ packets is greater than or equal to the number of servers $m$. If upon arrival a class $\beta$ packet finds all $m$ servers busy it may be queued if there are less than $K-m$ packets in the queue, otherwise it is lost. In this case, the threshold $K$ is selected such that the maximum (expected) delay of a class $\beta$ packet is less than an acceptable threshold $T^{\beta}_{max}$. The maximum delay of a class $\beta$ packet is when upon arrival it finds $K-1$ packets. It waits until all $K-m-1$ in front of it in the queue plus 1 packet in service are served at rate $m\mu$ and then it enters service. So, its expected waiting plus service time is $(K-m)/m\mu + 1/\mu \leq T^{\beta}_{max} \Rightarrow K \leq m\mu * T^{\beta}_{max}$.

The steady state probability, $\pi_n$ that there are $n$ packets in the system is

$$\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{n}\frac{\pi_0}{n!} \qquad n \leq m$$

$$\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{m\mu}\right)^{n-m}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{m}\frac{\pi_0}{m!} \qquad m \leq n \leq K$$

$$\left(\frac{\lambda^{\alpha}}{m\mu}\right)^{n-K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{m\mu}\right)^{K-m}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{m}\frac{\pi_0}{m!} \qquad K \leq n$$

In order to find the probability that the system is empty $\pi_0$, we use the fact that all system state probabilities sum to 1 to find:

$$\pi_0 = \left[\sum_{n=0}^{m-1}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{n}\frac{1}{n!} + \left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{m}\frac{1-\frac{\lambda^{\alpha}}{m\mu}-\frac{\lambda^{\beta}}{m\mu}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{m\mu}\right)^{K-m}}{m!\left(1-\frac{\lambda^{\alpha}}{m\mu}\right)\left(1-\frac{\lambda^{\alpha}+\lambda^{\beta}}{m\mu}\right)}\right]^{-1}$$

The probability that a class $\beta$ packet is lost is the probability that there are at least $K$ packets in the system :

$$P[n \geq K] = \left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{m\mu}\right)^{K-m}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{m}\frac{\pi_0}{m!\left(1-\frac{\lambda^{\alpha}}{m\mu}\right)}$$

From which we can find the average number of packets in the system, $\bar{N}$ (see [7] for details). The overall average packet delay is :

$$\bar{T} = \frac{\bar{N}}{\lambda^{\alpha}+\lambda^{\beta}(1-P[n \geq K])}$$

and the average packet delay for class $\alpha$ is :

$$\bar{T}^{\alpha} = \sum_{n=0}^{m-1}\frac{\pi_n}{\mu} + \sum_{n=m}^{\infty}\left(\frac{n-m+1}{m\mu}+\frac{1}{\mu}\right)\pi_n$$

In Appendix A, we present these performance measures for the special cases of $K=m$ and $m=1$.

### 2.2 $K \leq m$

Let us now consider the case where the blocking threshold $K$ for class $\beta$ packet is less than or equal to the number of servers $m$. If upon arrival a class $\beta$ packet finds less than $K$ servers busy, then it is starts been serviced, otherwise it is lost. In this case, the threshold $K$ can be selected such that the maximum blocking of a class $\beta$ packet is less than an acceptable threshold $B^{\beta}_{max}$, i.e. $P[n \geq K] \leq B^{\beta}_{max}$. Also, the service rate of any single server must be large enough to guarantee acceptable packet delay $1/\mu \leq T^{\beta}_{max}$.

Then the steady state probability of $n$ packets in the system, $\pi_n$, is:

$$\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{n}\frac{\pi_0}{n!} \qquad n \leq K$$

$$\left(\frac{\lambda^{\alpha}}{\mu}\right)^{n-K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\frac{\pi_0}{n!} \qquad K \leq n \leq m$$

$$\left(\frac{\lambda^{\alpha}}{m\mu}\right)^{n-m}\left(\frac{\lambda^{\alpha}}{\mu}\right)^{m-K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\frac{\pi_0}{m!} \qquad m \leq n$$

where,

$$\pi_0 = \left[\sum_{n=0}^{K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{n}\frac{1}{n!} + \left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\sum_{n=1}^{m-K}\left(\frac{\lambda^{\alpha}}{\mu}\right)^{n}\frac{1}{(K+n)!} + \left(\frac{\lambda^{\alpha}}{\mu}\right)^{m-K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\frac{1}{m!}\frac{\frac{\lambda^{\alpha}}{m\mu}}{1-\frac{\lambda^{\alpha}}{m\mu}}\right]^{-1}$$

The blocking probability for class $\beta$ is:

$$P[n \geq K] = \pi_0\left[\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\sum_{n=0}^{m-K}\left(\frac{\lambda^{\alpha}}{\mu}\right)^{n}\frac{1}{(K+n)!} + \left(\frac{\lambda^{\alpha}}{\mu}\right)^{m-K}\left(\frac{\lambda^{\alpha}+\lambda^{\beta}}{\mu}\right)^{K}\frac{1}{m!}\frac{\frac{\lambda^{\alpha}}{m\mu}}{1-\frac{\lambda^{\alpha}}{m\mu}}\right]$$

We can also find the average number in system, $\bar{N}$, (see [7] for details),

$$\bar{N} = \sum_{n=0}^{\infty} n\pi_n$$

the overall average packet delay,

$$\bar{T} = \frac{\bar{N}}{\lambda^{\alpha}+\lambda^{\beta}(1-P[n \geq K])}$$

and the class $\alpha$ average packet delay,

$$\bar{T}^{\alpha} = \sum_{n=0}^{m-1}\frac{1}{\mu}\pi_n + \sum_{n=m}^{\infty}\left(\frac{n-m+1}{m\mu}+\frac{1}{\mu}\right)\pi_n$$

In Appendix B, we give details for these performance measures for the special cases $K=m$ and $K=1$.

## 3 A Game Theory Formulation

In this section, we consider a routing problem in a parallel system that is composed of $L$ multi-server queueing systems, each of which operates like the one analyzed in the previous section.

Suppose class $c$ packets arrive to this parallel system at a rate $\lambda^c$ (Poisson arrivals) and that they may use any one of these $L$ multi-server queueing systems in order to reach a destination node. The fraction of class $c$ packets assigned to multi-server queueing system $i$ is $\phi_i^c$ and the vector of these fractions for each class $c$ is $\Phi^c = [\ldots\ \phi_i^c\ \ldots]$. There are $m_i$ servers at the queueing system $i$, each one with rate $\mu_i$ and the blocking threshold is $K_i$.

In this paper, for simplicity, we consider two classes of packets, i.e. $c \in \{\alpha, \beta\}$. Class $\alpha$ packets are queueable and therefore class $\alpha$ wants to minimize its average packet delay, while class $\beta$ packets are blocked and therefore class $\beta$ wants to minimize its blocking probability.

We formulate the problem as a *Nash* game between the two classes, where each class knows the cost functions and constraints for both classes. After reaching a Nash equilibrium, no class will have a rational motive to unilaterally deviate from its equilibrium strategy [1].

Class $\alpha$ solves the following problem:

*minimize* $\quad J^\alpha(\Phi^\alpha, \Phi^\beta) = \sum_{i=1}^{L} \phi_i^\alpha \bar{T}_i^\alpha$

*with respect to* $\quad \Phi^\alpha$

*such that* $\quad \sum_{i=1}^{L} \phi_i^\alpha = 1, \quad \phi_i^\alpha \ge 0 \ \ \forall\, i$

and class $\beta$ solves the following problem:

*minimize* $\quad J^\beta(\Phi^{\alpha *}, \Phi^\beta) = \sum_{i=1}^{L} \phi_i^\beta P[n_i \ge K_i]$

*with respect to* $\quad \Phi^\beta$

*such that* $\quad \sum_{i=1}^{L} \phi_i^\beta = 1, \quad \phi_i^\beta \ge 0 \ \ \forall\, i$

## 4 Example

Here, we consider a parallel system composed of two single-server queueing systems, and class $\beta$ packets are blocked when the server is busy, i.e. $L = 2, \ \ K = m = 1$.

Then the previously defined performance measures become :

$$\pi_{i,0} = \frac{\mu_i - \lambda_i^\alpha}{\mu_i + \lambda_i^\beta}$$

$$P[Blocking]_i^\beta = P[n_i \ge 1] = \frac{\lambda_i^\alpha + \lambda_i^\beta}{\mu_i + \lambda_i^\beta}$$

$$\bar{N}_i = \frac{(\lambda_i^\alpha + \lambda_i^\beta)\mu_i}{(\mu_i - \lambda_i^\alpha)(\mu_i + \lambda_i^\beta)}$$

$$\bar{T}_i = \frac{1}{\mu_i - \lambda_i^\alpha}$$

$$\bar{T}_i^\alpha = \frac{1}{\mu_i - \lambda_i^\alpha} + \frac{\lambda_i^\beta}{\mu_i(\mu_i + \lambda_i^\beta)}$$

If $\phi_i^c$ is the fraction of class $c$ packets routed to server $i$, then we can write $\lambda_i^c = \lambda^c \phi_i^c$ where $c \in \{\alpha, \beta\}$ and $i \in \{1, 2\}$.

Class $\alpha$ solves the following problem:

*minimize* $\quad J^\alpha(\Phi^\alpha, \Phi^{\beta *}) = \sum_{i=1}^{2} \phi_i^\alpha \left[ \frac{1}{\mu_i - \lambda^\alpha \phi_i^\alpha} + \frac{\lambda^\beta \phi_i^{\beta *}}{\mu_i(\mu_i + \lambda^\beta * \phi_i^{\beta *})} \right]$

*with respect to* $\quad \phi_1^\alpha,\ \phi_2^\alpha$

*such that* $\quad \sum_{i=1}^{2} \phi_i^\alpha = 1, \quad \phi_1^\alpha,\ \phi_2^\alpha \ge 0$

The first and second order derivatives of $J^\alpha$ with respect to $\phi_1^\alpha$ are

$$\frac{\partial J^\alpha}{\partial \phi_1^\alpha} = \frac{\mu_1}{(\mu_1 - \lambda^\alpha \phi_1^\alpha)^2} + \frac{\lambda^\beta * \phi_1^\beta}{\mu_1(\mu_1 + \lambda^\beta \phi_1^\beta)}$$

$$\frac{\partial^2 J^\alpha}{\partial (\phi_1^\alpha)^2} = \frac{2\mu_1 \lambda^\alpha}{(\mu_1 - \lambda^\alpha \phi_1^\alpha)^3}$$

and the cross derivative of $J^\alpha$ with respect to $\phi_1^\beta, \phi_1^\alpha$ is

$$\frac{\partial^2 J^\alpha}{\partial \phi_1^\beta \partial \phi_1^\alpha} = \frac{\lambda^\beta}{(\mu_1 + \lambda^\beta \phi_1^\beta)^2}$$

Similarly for the derivatives of $J^\alpha$ with respect to $\phi_2^\alpha$ and $\phi_2^\beta$.

Class $\beta$ solves the following problem:

*minimize* $\quad J^\beta(\Phi^{\alpha *}, \Phi^\beta) = \sum_{i=1}^{2} \phi_i^\beta \frac{(\lambda^\alpha \phi_i^{\alpha *} + \lambda^\beta \phi_i^\beta)}{\mu_i + \lambda^\beta \phi_i^\beta}$

*with respect to* $\quad \phi_1^\beta,\ \phi_2^\alpha$

*such that* $\quad \sum_{i=1}^{2} \phi_i^\beta = 1, \quad \phi_1^\beta,\ \phi_2^\alpha \ge 0$

The first and second order derivatives of $J^\beta$ with respect to $\phi_1^\beta$ are

$$\frac{\partial J^\beta}{\partial \phi_1^\beta} = 1 - \frac{\mu_1(\mu_1 - \lambda^\alpha \phi_1^\alpha)}{(\mu_1 + \lambda^\beta \phi_1^\beta)^3}$$

$$\frac{\partial^2 J^\beta}{\partial (\phi_1^\beta)^2} = \frac{2\lambda^\beta \mu_1(\mu_1 - \lambda^\alpha \phi_1^\alpha)}{(\mu_1 + \lambda^\beta \phi_1^\beta)^3}$$

and the cross derivative of $J^\beta$ with respect to $\phi_1^\alpha, \phi_1^\beta$ is

$$\frac{\partial^2 J^\beta}{\partial \phi_1^\alpha \partial \phi_1^\beta} = \frac{\lambda^\alpha \mu_1}{(\mu_1 + \lambda^\beta \phi_1^\beta)^2}$$

**Theorem:** *existence & uniqueness*

Let packets from two competing classes $\alpha$ and $\beta$ arrive according to Poisson distribution. They may be transmitted through multiple links with exponential service time distribution. Class $\alpha$ minimizes its average packet delay, while class $\beta$ minimizes its blocking probability. For the above routing game, there exists a unique Nash equilibrium solution.

**Proof:** The action spaces $\phi_1^\alpha + \phi_2^\alpha = 1$, $\phi_1^\alpha, \phi_2^\alpha \geq 0$ and $\phi_1^\beta + \phi_2^\beta = 1$, $\phi_1^\beta, \phi_2^\beta \geq 0$ define a convex, closed and compact set. The cost function $J^\alpha$ is jointly continuous in all its arguments and convex in $(\phi_1^\alpha, \phi_2^\alpha)$ for each fixed value of $(\phi_1^\beta, \phi_2^\beta)$. The cost function $J^\beta$ is jointly continuous in all its arguments and convex in $(\phi_1^\beta, \phi_2^\beta)$ for each fixed value of $(\phi_1^\alpha, \phi_2^\alpha)$. The function $J^\alpha + J^\beta$ is continuous and convex in $(\phi_1^\alpha, \phi_2^\alpha)$ for each fixed value of $(\phi_1^\beta, \phi_2^\beta)$ as well as is continuous and convex in $(\phi_1^\beta, \phi_2^\beta)$ for each fixed value of $(\phi_1^\alpha, \phi_2^\alpha)$. Therefore the above routing game admits a Nash equilibrium.

The Jacobian matrix with elements $\partial^2 J^c / \partial\phi_j^c \partial\phi_i^k$, $c, k = \alpha, \beta$, $i, j = 1, 2$ is strictly diagonally dominant for all $(\phi_1^\alpha, \phi_2^\alpha, \phi_1^\beta, \phi_2^\beta)$ such that $\phi_1^\alpha + \phi_2^\alpha = 1$, $\phi_1^\beta + \phi_2^\beta = 1$, $\phi_1^\alpha, \phi_2^\alpha, \phi_1^\beta, \phi_2^\beta \geq 0$, $C_1 - \lambda^\alpha\phi_1^\alpha - \lambda^\beta\phi_1^\beta > 0$ and $C_2 - \lambda^\alpha\phi_2^\alpha - \lambda^\beta\phi_2^\beta > 0$.□

The following policy gives the Nash equilibrium solution where class $\alpha$ minimizes its average packet delay, while class $\beta$ minimizes its blocking probability :

$$If \quad \lambda^\alpha < \mu_1 - \sqrt{\frac{\mu_1\mu_2(\mu_2 + \lambda^\beta)}{\mu_2 + 2\lambda^\beta}}$$

$$and \quad \lambda^\beta \leq \mu_2\sqrt{\frac{\mu_1}{\mu_1 - \lambda^\alpha}} - \mu_2$$

$$then \quad \phi_1^{\alpha*} = 1, \quad \phi^{\beta*} = 0$$

$$If \quad \lambda^\alpha < \mu_2 - \sqrt{\frac{\mu_1\mu_2(\mu_1 + \lambda^\beta)}{\mu_1 + 2\lambda^\beta}}$$

$$and \quad \lambda^\beta \leq \mu_1\sqrt{\frac{\mu_2}{\mu_2 - \lambda^\alpha}} - \mu_1$$

$$then \quad \phi_1^{\alpha*} = 0, \quad \phi^{\beta*} = 1$$

$$If \quad \lambda^\alpha < \mu_1, \lambda^\beta \geq \sqrt{\mu_1(\mu_1 - \lambda^\alpha)} - \mu_1$$

$$and \quad \lambda^\beta \geq \mu_2\sqrt{\frac{\mu_1}{\mu_1 - \lambda^\alpha}} - \mu_2$$

$$then \quad \phi_1^{\alpha*} = 1$$

$$\phi_1^{\beta*} = -\frac{\mu_1}{\lambda^\beta} + \frac{\mu_1 + \mu_2 + \lambda^\beta}{\lambda^\beta}\frac{\sqrt{\mu_1(\mu_1 - \lambda^\alpha)}}{\sqrt{\mu_1(\mu_1 - \lambda^\alpha)} + \mu_2}$$

accept the solution only if $\lambda^\alpha \leq \mu_1 -$

$$\sqrt{\frac{\mu_1}{\frac{1}{\mu_2} + \frac{\lambda^\beta\phi_2^{\beta*}}{\mu_2(\mu_2 + \lambda^\beta\phi_2^{\beta*})} - \frac{\lambda^\beta\phi_1^{\beta*}}{\mu_1(\mu_1 + \lambda^\beta\phi_1^{\beta*})}}}$$

$$If \quad \lambda^\alpha < \mu_2, \lambda^\beta \geq \sqrt{\mu_2(\mu_2 - \lambda^\alpha)} - \mu_2$$

$$and \quad \lambda^\beta \geq \mu_1\sqrt{\frac{\mu_2}{\mu_2 - \lambda^\alpha}} - \mu_1$$

$$then \quad \phi_1^{\alpha*} = 0,$$

$$\phi_1^{\beta*} = \frac{\mu_2 + \lambda^\beta}{\lambda^\beta}\frac{\mu_1}{\mu_1 + \sqrt{\mu_2(\mu_2 - \lambda^\alpha)}}$$

accept the solution only if $\lambda^\alpha \leq \mu_2 -$

$$\sqrt{\frac{\mu_2}{\frac{1}{\mu_1} + \frac{\lambda^\beta\phi_1^{\beta*}}{\mu_1(\mu_1 + \lambda^\beta\phi_1^{\beta*})} - \frac{\lambda^\beta\phi_2^{\beta*}}{\mu_2(\mu_2 + \lambda^\beta\phi_2^{\beta*})}}}$$

$$If \quad \mu_1(\mu_1 + \lambda^\beta) > \mu_2\lambda^\beta$$

$$\lambda^\alpha \geq \mu_1 - \mu_1\sqrt{\frac{\mu_2(\mu_1 + \lambda^\beta)}{\mu_1(\mu_1 + \lambda^\beta) - \mu_2\lambda^\beta}}$$

$$and \quad \lambda^\alpha \geq \mu_2 - \sqrt{\frac{\mu_1\mu_2(\mu_1 + \lambda^\beta)}{\mu_1 + 2\lambda^\beta}}$$

$$then \quad \phi_1^{\alpha*} = arg\min_{\phi_1^\alpha \geq 0} J^\alpha(\phi_1^\alpha, 1)$$

$$\phi_1^{\beta*} = 1$$

accept the solution only if

$$\mu_1 - \lambda^\alpha\phi_1^{\alpha*} > 0, \quad \mu_2 - \lambda^\alpha\phi_2^{\alpha*} > 0,$$

$$\lambda^\beta \leq \sqrt{\frac{\mu_1\mu_2(\mu_1 - \lambda^\alpha\phi_1^{\alpha*})}{\mu_2 - \lambda^\alpha\phi_2^{\alpha*}}} - \mu_1$$

$$If \quad \mu_2(\mu_2 + \lambda^\beta) > \mu_1\lambda^\beta$$

$$\lambda^\alpha \geq \mu_1 - \sqrt{\frac{\mu_1\mu_2(\mu_2 + \lambda^\beta)}{\mu_2 + 2\lambda^\beta}}$$

$$and \quad \lambda^\alpha \geq \mu_2 - \mu_2\sqrt{\frac{\mu_1(\mu_2 + \lambda^\beta)}{\mu_2(\mu_2 + \lambda^\beta) - \mu_1\lambda^\beta}}$$

$$then \quad \phi_1^{\alpha*} = arg\min_{\phi_1^\alpha \geq 0} J^\alpha(\phi_1^\alpha, 0)$$

$$\phi_1^{\beta*} = 0$$

accept the solution only if

$$\mu_1 - \lambda^\alpha\phi_1^{\alpha*} > 0, \quad \mu_2 - \lambda^\alpha\phi_2^{\alpha*} > 0,$$

$$\lambda^\beta \leq \sqrt{\frac{\mu_1\mu_2(\mu_2 - \lambda^\alpha\phi_2^{\alpha*})}{\mu_1 - \lambda^\alpha\phi_1^{\alpha*}}} - \mu_2$$

For all other cases:

$$\phi_1^{\alpha*} = arg\min_{\phi_1^\alpha \geq 0} J^\alpha(\phi_1^\alpha, \phi_1^{\beta*})$$

$$\phi_1^{\beta*} = arg\min_{\phi_1^\beta \geq 0} J^\beta(\phi_1^{\alpha*}, \phi_1^\beta)$$

Note that in the above policy the cases $\phi_1^{\alpha*} = \phi_1^{\beta*} = 1$ and $\phi_1^{\alpha*} = \phi_1^{\beta*} = 0$ do not appear. The following Corollary follows.

**Corollary:** Let packets, with exponentially distributed length, from two competing classes $\alpha$ and $\beta$ arrive according to a Poisson distribution. They are transmitted over multiple links. Class $\alpha$ minimizes its average packet delay, while class $\beta$ minimizes its blocking probability. For the above routing game, it is never the case that both classes use the same link exclusively.

In Figure 2, we show the Nash equilibrium fractions, $\phi_1^{\alpha*}$ and $\phi_1^{\beta*}$, for equal arrival rates $\lambda^\alpha = \lambda^\beta$ and unequal service rates, $\mu_1 = 2$, $\mu_2 = 1$. We notice that for light load, class $\alpha$ uses the faster server, while class $\beta$ uses both servers. As the load increases, both classes use both servers.

In Figure 3, the Nash equilibrium fractions, $\phi_1^{\alpha*}$ and $\phi_1^{\beta*}$, are plotted for a fixed class $\beta$ arrival rate $\lambda^\beta = 1$ and service rates, $\mu_1 = 2$, $\mu_2 = 1$. We notice a behavior similar to that found in Figure 2.

In Figure 4, we consider a similar scenario except that the arrival rate for class $\beta$ has ben reduced by a factor of ten, i.e., $\lambda^\beta = 0.1$. For this case class $\alpha$ uses the faster server while class $\beta$ uses both servers. As the load increases, class $\alpha$ starts using both servers while class $\beta$ uses only the faster server.

We further explore this last case of small class $\beta$ arrival rate and exaggerate the difference in service rates between the servers. In Figure 5, we consider the case where $\mu_1 = 10$ and $\mu_2 = 1$. The behavior is similar to Figure 4. For light load, class $\alpha$ uses the faster server exclusively, while class $\beta$ uses both servers but with preference for the faster server. As the load increases, class $\alpha$ continues to use the faster server, while class $\beta$ turns to the slower server and uses it without interference from class $\alpha$. However, for very heavy load, class $\alpha$ starts using both servers and as shown in Figure 7, this has an immediate impact on the blocking probability of class $\beta$. In Figure 6, the class $\alpha$ mean dealy is plotted and in Figure 7, we plot the class $\beta$ blocking probability for this case. As noted, for intermediate load there is separation between the classes and the blocking probability of class $\beta$ is independent of the class $\alpha$ load. For high values of $\lambda^\alpha$, class $\alpha$ starts to use both servers and class $\beta$ suffers more blocking.

## 5 Conclusions

In this paper, we have considered the routing problem in a network providing integrated transport, where one class of packets wants to minimize its average packet delay, while another class of packets wants to minimize its blocking probability. We modeled and found several performance measures for a multi-server queueing system, where packets of the first call can be queued, while packets of the other class are blocked when the number of packets in the system is more than some threshold.

We then considered the routing problem through a parallel system of such multi-server queues with blocking. We formulated the problem as a Nash game between the two classes and found the Nash equilibrium solution.

Extensions of this work may be to consider an arbitrary network with multiple classes, where the classes having different blocking thresholds and mean service requirements. We are also investigating a threshold based on delay rather than queue size, this seems to be more appropriate for different rate servers and is modelled by different threshold at the different servers.

## A Special Case

i) $K = m$

In this case class $\beta$ packets can not be queued. So, if upon arrival a class $\beta$ packet finds all $m$ servers busy it is lost. Then the previously defined performance measures become :

$$\pi_0 = \left[\sum_{n=0}^{m-1}\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^n\frac{1}{n!} + \left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^m\frac{1}{m!}\frac{1}{1-\frac{\lambda^\alpha}{m\mu}}\right]^{-1}$$

$$P[n\geq m] = \left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^m\frac{1}{m!}\frac{\pi_0}{1-\frac{\lambda^\alpha}{m\mu}}$$

$$\bar{N} = \pi_0\left\{\sum_{n=1}^{m-1}\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^n\frac{1}{(n-1)!} + \left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^m\frac{1}{m!}\left[\frac{m}{1-\frac{\lambda^\alpha}{m\mu}} + \frac{\frac{\lambda^\alpha}{m\mu}}{\left(1-\frac{\lambda^\alpha}{m\mu}\right)^2}\right]\right\}$$

$$\bar{T} = \frac{\bar{N}}{\lambda^\alpha + \lambda^\beta(1-P[n\geq m])}$$

$$\bar{T}^\alpha = \frac{\pi_0}{\mu}\left\{\sum_{n=0}^{m-1}\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^n\frac{1}{n!} + \left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^m\frac{1}{m!}\left[\frac{1}{1-\frac{\lambda^\alpha}{m\mu}} + \frac{\frac{1}{m}}{\left(1-\frac{\lambda^\alpha}{m\mu}\right)^2}\right]\right\}$$

ii) $m = 1$

In this case, there is only one server. Then the previously defined performance measures become :

$$\pi_0 = \frac{1-\frac{\lambda^\alpha}{\mu}-\frac{\lambda^\beta}{\mu}\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^K}{\left(1-\frac{\lambda^\alpha}{\mu}\right)\left(1-\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)}$$

$$P[n\geq K] = \frac{\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^K\left(1-\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)}{1-\frac{\lambda^\alpha}{\mu}-\frac{\lambda^\beta}{\mu}\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^K}$$

$$\bar{N} = \pi_0\left\{\frac{\lambda^\alpha+\lambda^\beta}{\mu}\frac{1-(K+1)\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^K + K\left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^{K+1}}{\left(1-\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^2} + \left(\frac{\lambda^\alpha+\lambda^\beta}{\mu}\right)^K\left[K\frac{\frac{\lambda^\alpha}{\mu}}{1-\frac{\lambda^\alpha}{\mu}} + \frac{\frac{\lambda^\alpha}{\mu}}{\left(1-\frac{\lambda^\alpha}{\mu}\right)^2}\right]\right\}$$

$$\bar{T} = \frac{\bar{N}}{\lambda^\alpha + \lambda^\beta(1 - P[n \geq K])}$$

$$\bar{T}^\alpha = \frac{\pi_0}{\mu}\left\{1 + \frac{\lambda^\alpha + \lambda^\beta}{\mu}\left[\frac{1 - \left(\frac{\lambda^\alpha + \lambda^\beta}{\mu}\right)^{K-1}}{1 - \frac{\lambda^\alpha + \lambda^\beta}{\mu}} + \right.\right.$$

$$\left.\frac{1 - K\left(\frac{\lambda^\alpha + \lambda^\beta}{\mu}\right)^{K-1} + (K-1)\left(\frac{\lambda^\alpha + \lambda^\beta}{\mu}\right)^{K}}{\left(1 - \frac{\lambda^\alpha + \lambda^\beta}{\mu}\right)^2}\right] +$$

$$\left. + \left(\frac{\lambda^\alpha + \lambda^\beta}{\mu}\right)^K \left[\frac{K}{1 - \frac{\lambda^\alpha}{\mu}} + \frac{1}{\left(1 - \frac{\lambda^\alpha}{\mu}\right)^2}\right]\right\}$$

## B Special Cases

i) $K = m$ same as in Appendix A.
ii) $K = 1$

In this case, class $\beta$ packets may use only one server. A class $\beta$ packet arrival that finds another class $\beta$ packet in service is lost. Then the previously defined performance measures become :

$$\pi_0 = \left[1 + \frac{\lambda^\alpha + \lambda^\beta}{\mu}\sum_{n=0}^{m-1}\left(\frac{\lambda^\alpha}{\mu}\right)^n \frac{1}{(n+1)!} + \right.$$

$$\left.\left(\frac{\lambda^\alpha}{\mu}\right)^{m-1} \frac{\lambda^\alpha + \lambda^\beta}{\mu}\frac{1}{m!}\frac{\frac{\lambda^\alpha}{m\mu}}{1 - \frac{\lambda^\alpha}{m\mu}}\right]^{-1}$$

$$P[n \geq 1] = 1 - \pi_0$$

$$\bar{N} = \pi_0\left\{\frac{\lambda^\alpha + \lambda^\beta}{\mu}\sum_{n=0}^{m-1}\left(\frac{\lambda^\alpha}{\mu}\right)^n \frac{1}{n!} + \right.$$

$$\left.\left(\frac{\lambda^\alpha}{\mu}\right)^{m-1}\frac{\lambda^\alpha + \lambda^\beta}{\mu}\frac{1}{m!}\left[m\frac{\frac{\lambda^\alpha}{m\mu}}{1 - \frac{\lambda^\alpha}{m\mu}} + \frac{\frac{\lambda^\alpha}{m\mu}}{\left(1 - \frac{\lambda^\alpha}{m\mu}\right)^2}\right]\right\}$$

$$\bar{T} = \frac{\bar{N}}{\lambda^\alpha + \lambda^\beta \pi_0}$$

$$\bar{T}^\alpha = \frac{\pi_0}{\mu}\left\{1 + \frac{\lambda^\alpha + \lambda^\beta}{\mu}\sum_{n=0}^{m-2}\left(\frac{\lambda^\alpha}{\mu}\right)^n \frac{1}{(n+1)!}\right.$$

$$\left. + \left(\frac{\lambda^\alpha}{\mu}\right)^{m-1}\frac{\lambda^\alpha + \lambda^\beta}{\mu}\frac{1}{m!}\left[\frac{1}{1 - \frac{\lambda^\alpha}{m\mu}} + \frac{\frac{1}{m}}{\left(1 - \frac{\lambda^\alpha}{m\mu}\right)^2}\right]\right\}$$

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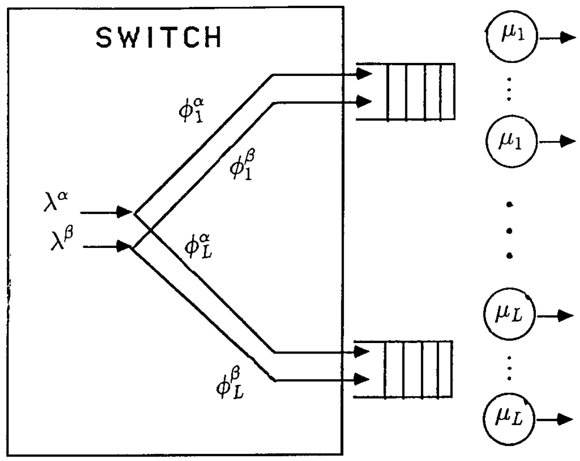


Figure 1: Routing in a Switch

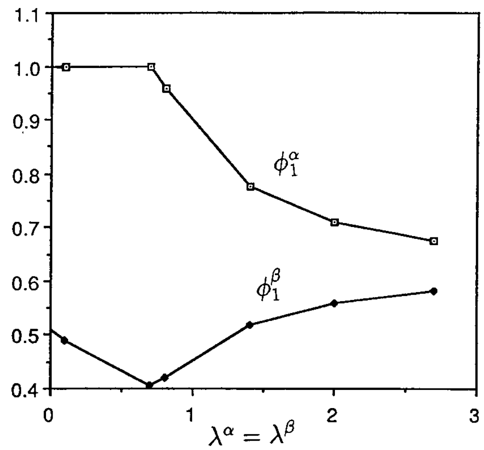


Figure 2: Nash equilibrium routing fractions for equal arrival rates $\lambda^\alpha = \lambda^\beta$ and unequal server rates $\mu_1 = 2$, $\mu_2 = 1$

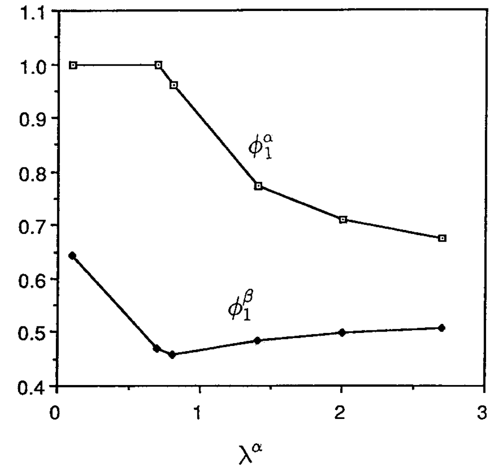


Figure 3: Nash equilibrium routing fractions for fixed class $\beta$ arrival rate $\lambda^\beta = 1$ and varying class $\alpha$, unequal server rates $\mu_1 = 2$, $\mu_2 = 1$

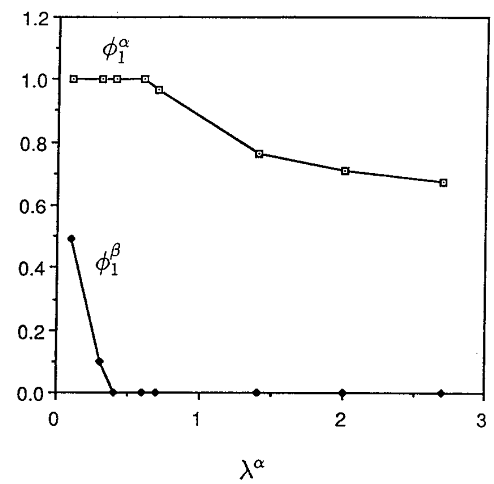


Figure 4: Nash equilibrium routing fractions for smaller fixed class $\beta$ arrival rate $\lambda^\beta = 0.1$ and varying class $\alpha$, unequal server rates $\mu_1 = 2$, $\mu_2 = 1$

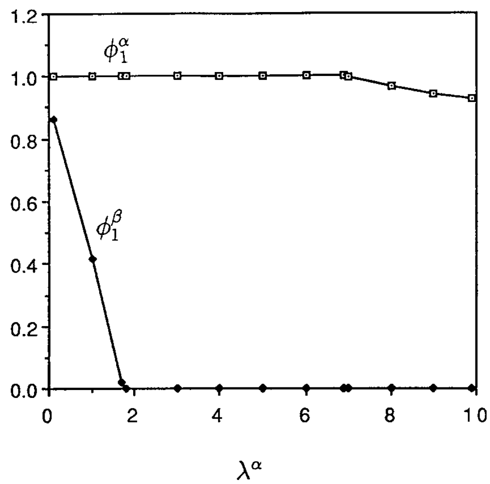


Figure 5: Nash equilibrium routing fractions for fixed class $\beta$ arrival rate $\lambda^\beta = 0.1$ and varying class $\alpha$, more unequal server rates $\mu_1 = 10$, $\mu_2 = 1$

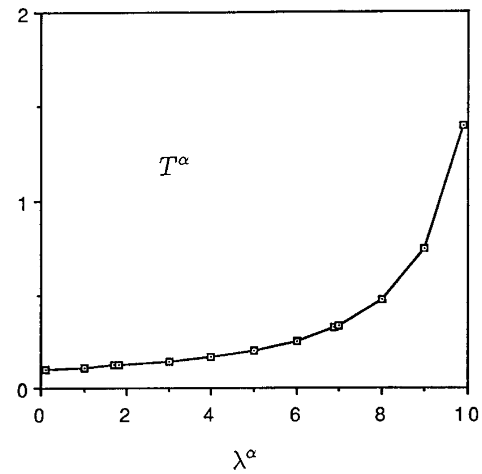


Figure 6: Class $\alpha$ mean delay for fixed class $\beta$ arrival rate $\lambda^\beta = 0.1$ and varying class $\alpha$, more unequal server rates $\mu_1 = 10$, $\mu_2 = 1$

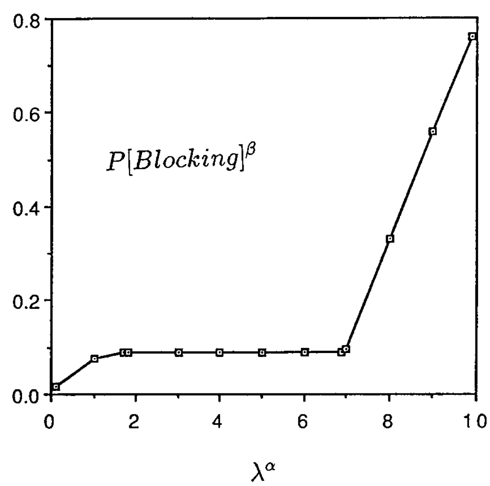


Figure 7: Class $\beta$ blocking probability for fixed class $\beta$ arrival rate $\lambda^\beta = 0.1$ and varying class $\alpha$, more unequal server rates $\mu_1 = 10$, $\mu_2 = 1$