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# ADAPTIVE ROUTING & CONGESTION CONTROL FOR WINDOW FLOW CONTROLLED VIRTUAL CIRCUIT NETWORKS

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**Abstract**

Routing, congestion, flow and error control are important problems for efficient network operation. Previous studies on virtual circuit network management and control consider each problem separately. However, all these problems are strongly interrelated and each affects the others. In this paper, all the above problems are simultaneously solved to improve virtual circuit network performance.

In contrast to previous studies, a state space nonlinear nonstationary queueing model for dynamic virtual circuit networks that considers the dynamic interaction among the virtual circuit and packet processes is introduced. Optimal virtual circuit congestion control, routing decisions and flow control window sizes are derived. The combined cost of maintaining virtual circuits and the cost of packet delays are minimized while the profit of admitting new virtual circuits and the profit of packet throughput are maximized.

## 1. INTRODUCTION

The key network management algorithms are error control, routing, flow control and congestion control. *Error control* ensures reliable packet delivery at the destination [1, 22]. *Routing* decides which route the message will follow from source to destination [15, 1, 22]. This important algorithm has received considerable attention for datagram networks and only recently for virtual circuit networks [11, 1, 22]. *Congestion control* prevents network overload by controling the traffic entering the network. *Flow control* prevents a source-destination pair from monopolizing the network and a fast sender from oversaturating a slow receiver, by controlling their traffic [15, 1, 22].

Error control, routing, congestion and flow control are strongly related problems and each one affects the others. For a more accurate network model and better network performance, all of the above problems should be modeled and solved simultaneously. Such an approach however may increase the modeling and optimization complexity. Previous studies on virtual circuit network management consider at most one of the key network management problems. However, if one of the network controls is ignored, then the "optimal" network operation will be far from optimality, as we have shown in [4] for datagram networks. In this paper, we consider the combined problem of routing and congestion control for dynamic virtual circuit networks, where the effects of the flow and error control mechanisms are included.

In virtual circuit networks, the first packet of a virtual circuit establishes a path from source to destination that remains fixed for the duration of the call and all packets belonging to this virtual circuit follow this path. Codex, Euronet, SNA, Telenet, Transpac and Tymnet (among others) are existing networks employing virtual circuit switching [11, 1, 22].

Some previous studies on the virtual circuit routing problem are the following :

Segall [25] formulates the virtual circuit routing problem as a nonlinear programming problem on the link flows space. A virtual circuit is routed through the node with the minimum incremental delay to the destination.

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Gerla & Nilsson [12] analyze a network with a fixed number of virtual circuits, each one with window flow control. After modeling each virtual circuit as a closed chain, they approximate it with an open chain. They propose routing along the minimum first derivative length path.

Lam & Lien [16, 17] investigate the routing of a new virtual circuit with only one packet in a network with a fixed number of virtual circuits, such that the increase in the mean network delay is minimized.

Gersht [13] considers a dynamic virtual circuit routing algorithm. At each node, the available paths for routing to a destination node are the minimum hop paths to that destination node. A virtual circuit is routed through the outgoing link with the current minimum number of virtual circuits.

Gafni & Bertsekas [9, 10] formulate the virtual circuit routing problem on the path flows space. A virtual circuit is routed along the shortest marginal delay path.

Tsitsiklis & Bertsekas [29], Tsai, Tsitsiklis & Bertsekas [28] and Tsai [27] use the gradient projection method for the asynchronous virtual circuit routing problem on the path flows space and the routing decisions try to achieve some optimal target flows. They suggest that deterministic routing is better than randomized routing.

Economides, Ioannou & Silvester [3] use stochastic learning automata to adaptively route virtual circuits along the currently minimum unfinished work path, for user optimization, or along the path with the minimum increase in the number of packets or in the proportional packet delay, for system optimization.

Tipper & Sundareshan [26] suggest routing a new virtual circuit along the shortest path whose utilization does not exceed its threshold in order to minimize the average packet delay.

Some previous studies on the window flow control of a virtual circuit are the following:

Reiser [19, 20] models a virtual circuit with window flow control both as a closed chain and as an open chain, that also considers the packet delay before the packet enters the network. He also finds window sizes to maximize the power.

Schwartz [23, 22] analyzes several window flow control mechanisms for a virtual circuit that is represented by a closed chain and he proposes window sizes that maximize the ratio of the throughput over the average packet delay (power) for a virtual circuit.

In virtual circuits there are two dependent processes, the virtual circuit process and the packet process, that occur at different time scales. Previous work on virtual circuit networks do not explicitly consider the dynamic interaction between these processes. In [3], we modeled the interaction among the virtual circuit and packet processes using steady state queueing models. Now, in this paper we use a state space approach to describe both the virtual circuit and the packet processes simultaneously using nonlinear non-stationary queueing models. The network dynamics occur at two different time scales. The virtual circuit process evolves at the slower time scale and is used in the evolution of the packet process that happens at the faster time scale.

We define a nonlinear non-stationary queueing model for dynamic virtual circuit networks using a state space approach and we formulate the integrated routing, congestion, flow and error control problem for virtual circuit networks as an optimal control problem. A state space approach has also been taken for the routing problem in datagram networks [24, 18, 5, 6, 7, 14, 21]. For the routing problem in virtual circuit networks, Tipper & Sundareshan [26] use a non linear state space model for the average number of packets on each link. Their objective is to minimize the average packet delay.

We set up a multiobjective function to be optimized and we derive optimal virtual circuit routing decisions and optimal congestion controls. The combined cost of maintaining virtual circuits and the cost of packet delays are minimized while the profit of admitting new virtual circuits and the profit of packet throughput are maximized.

## 2. DYNAMIC VIRTUAL CIRCUIT NETWORK MODEL

We first consider a simple one hop network with multiple links. Virtual circuits arrive at a node $s$ (according to a Poisson distribution) destined for another node $d$ with rate $\gamma(t)$. A virtual circuit is admitted into the network with probability $\phi(t) \in [0,1]$ at time $t$, only if the increase in the cost that it will cause is less than the increase in the profit that it will supply to the network management, otherwise it is rejected with probability $1-\phi(t)$. An admission controller makes these decisions and updates this admission probability $\phi(t)$ according to the current network state. Then the actual virtual circuit arrival rate into the network is $\gamma(t) * \phi(t)$.

If the virtual circuit is accepted, then it is routed to node $d$ through one of the $L$ links that connect these two nodes with probability $P_i(t) \in [0,1]$ through link $i$, $i = 1, ..., L$, $\sum_i P_i(t) = 1$ at time $t$. A router makes these decisions and updates these routing probabilities according to the current network state. Then the virtual circuit arrival rate to link $i$ is $\gamma(t) * \phi(t) * P_i(t)$.

Finally, each virtual circuit stays in the network for some time duration generally distributed with mean $1/\delta(t)$ and then terminates. So, we can model every link $i$ for the virtual circuit process as an $M/G/\infty$ queue with arrival rate $\gamma(t) * \phi(t) * P_i(t)$ and mean service time $1/\delta(t)$ (Fig. 1). Subsequently, we will introduce a state space approach to model the dynamic evolution of the virtual circuit processes.

The expected number of virtual circuits on link $i$ at time $t$, $V_i(t) \geq 0$, $i = 1, ...L$ increases during $\Delta t$ by the expected number of virtual circuits that arrive during this period, $\gamma(t) * \phi(t) * P_i(t) * \Delta t$, minus the expected number of virtual circuits that depart during this period, $\delta(t) * V_i(t) * \Delta t$. So, the virtual circuit process at link $i$ is described by

$$\begin{aligned} V_i(t+\Delta t) &= V_i(t) + \gamma(t) * \phi(t) * P_i(t) * \Delta t - \delta(t) * V_i(t) * \Delta t \qquad i = 1, ..., L \quad or \\ \dot{V}_i(t) &= \gamma(t) * \phi(t) * P_i(t) - \delta(t) * V_i(t) \quad for \ \ \Delta t \to 0, \qquad i = 1, ..., L \end{aligned}$$

Next we describe the evolution of the packet process into the network. Let $r(t)$ be the packet arrival rate per virtual circuit at time $t$ (Poisson distribution). If there are $V_i(t)$ virtual circuits on link $i$ at time $t$, then the total packet arrival rate to link $i$ is $r(t) * V_i(t)$, since all packets belonging to a virtual circuit are transmitted through the same link.

Upon arriving to link $i$, a packet is not immediately accepted for transmission, but first stays in the input queue for link $i$ (Fig. 2), where the expected number of packets at time $t$ is $I_i(t) \geq 0$, $i = 1, ..., L$. A window flow control mechanism permits at most $W_i(t)$ packets to exist on link $i$ (either in the transmission queue or being actually transmitted) at time $t$, where $W_i(t) \geq 0$, $i = 1, ..., L$. A packet that is to be transmitted through link $i$ waits in the input queue for this link until a permit comes back from a successfully received packet at node $d$. Let $A_i(t)$ be the expected number of permits available at link $i$ at time $t$, $A_i(t) \geq 0$, $i = 1, ..., L$.

Let the expected number of packets for link $i$, $N_i(t) \geq 0$, $i = 1, ..., L$, be equal to the expected number of packets in the input queue for link $i$, $I_i(t)$, plus the expected number of packets in transit at link $i$, i.e. the window size $W_i(t)$, minus the expected number of available permits $A_i(t)$

$$N_i(t) = I_i(t) + W_i(t) - A_i(t) \qquad i = 1, ..., L$$

When a permit becomes available, the packet is accepted into the transmission queue for link $i$ and waits for its transmission (on first-come first-served scheduling). When reaching the head of this queue, the packet is transmitted. Let $\tau_i$ be the mean packet transmission time at link $i$ (general distribution) and $\eta_i$ be the packet propagation delay at link $i$ (general distribution).

However, with probability $e_i(t)$ for link $i$ at time $t$, $e_i(t) \in [0,1]$, the packet may arrive in error (or perhaps never arrive) at node $d$. If the packet arrives in error, node $s$ is notified with a negative acknowledgment (NACK) by node $d$ and let $\theta_i$ be the NACK/time-out delay for link $i$ (general distribution). Then node $s$ retransmits it. If it never arrives, eventually node $s$ times out after time $\theta_i$ and retransmits it.

If a packet successfully arrives at node $d$, a positive acknowledgment (ACK) is sent back to node $s$ where it arrives after time $\alpha_i$ (general distribution). This ACK also acts as a permit for the transmission of another packet through this link. However, the ACK may also fail, with probability $e_i'(t) \in [0,1]$, and therefore node $s$ will retransmit the packet after time $\beta_i$ (general distribution).

Next, we find the mean time $x_i(t)$ that a packet spends on link $i$ after starting transmission (no queueing delays) at time $t$ (Fig. 2 & 3). This will be the sum of the mean transmission delay $\tau_i$, plus the propagation delay $\eta_i$, plus, with error probability $e_i(t)$, the mean time spent when there is a packet error, plus, with no error probability $1 - e_i(t)$, the mean time spent for the ACK. The mean time spent when there is a packet error is the sum of the mean NACK/time-out delay plus the mean retransmission time $x_i(t)$. The mean time spent for the ACK is the sum of the mean ACK delay $\alpha_i$ plus the mean NACK/time-out delay $\beta_i(t)$ and mean retransmission time $x_i(t)$ in case the ACK fails with probability $e_i'(t)$

$$x_i(t) = \tau_i + \eta_i + e_i(t) * (\theta_i + x_i(t)) + (1 - e_i(t)) * [\alpha_i + e_i'(t) * (\beta_i(t) + x_i(t))]$$

Next, we assume that the packet service times on link $i$ are independent exponentially distributed random variables with rate $\mu_i(t) = 1/x_i(t)$. Thus:

$$\mu_i(t) = \frac{1 - e_i(t) - [1 - e_i(t) * e_i'(t)]}{\tau_i + \eta_i + e_i(t) * \theta_i + [1 - e_i(t)] * [\alpha_i + e_i'(t) * \beta_i]}$$

In general, it is not true that the packet service times will be independent exponentially distributed, but we make this assumption, in order to have a simple model that captures the effect of the link error rate and ACK process on the packet service time.

Then the Markov chain with state $N_i = I_i + W_i - A_i$ is an $M/M/1$ queue (Fig. 3 & 4). So, for the packet process, we model each link $i$ as an $M/M/1$ queue, with packet arrival rate $r(t) * V_i(t)$ and mean service time $1/\mu_i(t)$.

Let $\rho_i(N_i(t))$ be an approximation of the instantaneous utilization for link $i$ such that the packet departure rate from link $i$ at time $t$ is $\mu_i(t) * \rho_i(N_i(t))$.

Then the expected number of packets at link $i$ at time $t$, $N_i(t)$, will increase during $\Delta t$ by the expected number of packets that will arrive during this period, $r(t) * V_i(t) * \Delta t$, minus the expected number of packets that will depart during this period, $\mu_i(t) * \rho_i(t)$. Since, the link utilization $\rho_i(N_i(t))$, is a nonlinear function of the number of packets at link $i$, $N_i(t)$, the packet process at link $i$ is described by a non linear dynamic model

$$\begin{aligned} N_i(t + \Delta t) &= N_i(t) + r(t) * V_i(t) * \Delta t - \mu_i(t) * \rho_i(N_i(t)) * \Delta t \quad i = 1, ..., L \quad or \\ \dot{N}_i(t) &= r(t) * V_i(t) - \mu_i(t) * \rho_i(N_i(t)) \quad for \;\; \Delta t \to 0, \quad i = 1, ..., L \end{aligned}$$

A similar nonlinear non stationary model for the average number of packets at a link has also been considered by Filipiak [5, 6, 7, 8] and by Tipper & Sundareshan [26]. However, in our model we make the packet arrival rate depend on the expected number of virtual circuits, i.e. the network state. In this way, we introduce the dynamic interaction among the virtual circuit and packet processes into our model. Although a nonlinear model will introduce extra complexity in the optimization procedures, we prefer to have a nonlinear model than a simpler linear model, since the queueing processes are extremely nonlinear processes.

Having described the dynamics of the virtual circuit and the packet processes, we define the network state to be

$$\mathbf{X}(t) = [V_1(t), ..., V_L(t), N_1(t), ..., N_L(t)]^T$$

In the next section, we will use this non linear dynamic model for non stationary virtual circuit networks to formulate and solve the combined virtual circuit routing, congestion contol and window flow control problem as an optimal control problem. Note, also that the effect of the variable link error rates has been included into the model.

## 3. MULTI - OBJECTIVE FUNCTION

In this section, we formulate the integrated routing, congestion, window flow and error control problem for virtual circuit networks as an optimal control problem. An optimal control approach has also been taken by Filipiak [5, 8], for the routing and congestion control problems, for datagram networks. He minimized the average packet delay and maximized the gain due to acceptance of packets into the network. Also, Tipper & Sundareshan [26] formulated the routing problem for virtual circuit networks as an optimal control problem. Their objective function was also to minimize the average packet delay.

Here, we set up a multiobjective function to be optimized, for the integrated problem. We would like to minimize the cost of setting up and maintaining the virtual circuits inside the network, while also maximizing the profit of admitting virtual circuits into the network. We would also like to minimize the cost of having the packets inside the network, while also maximizing the profit of servicing packets. To accomplish this, we define the following costs and profits :

$C_{V,i}(t)$ : cost per virtual circuit for link $i$ at time $t$, (for example, the cost of setting up and maintaining the virtual circuit path through link $i$).

$C_{\phi}(t)$ : profit of admitting a new virtual circuit into the network at time $t$, or cost of rejecting a new virtual circuit at time $t$.

$C_{N,i}(t)$ : cost per packet for link $i$ at time $t$.

$C_{\mu,i}(t)$ : profit of servicing a packet at link $i$ at time $t$.

Then our problem is to

*minimize*
$$\int_0^T \{ \sum_i C_{V,i}(t) * V_i(t) - C_{\phi}(t) * \phi(t) * \gamma(t) + + \sum_i C_{N,i}(t) * N_i(t) - \sum_i C_{\mu,i}(t) * \mu_i(t) * \rho_i(t) \} dt$$

*with respect to* the routing probabilities $P_i(t)$ $i = 1, ..., L$, the congestion control parameter $\phi(t)$

*such that*

$$\dot{\mathbf{X}}(t) = \begin{bmatrix} \gamma(t) * \phi(t) * P_1(t) - \delta(t) * V_1(t) \\ \gamma(t) * \phi(t) * P_2(t) - \delta(t) * V_2(t) \\ \dots \\ \gamma(t) * \phi(t) * P_L(t) - \delta(t) * V_L(t) \\ r(t) * V_1(t) - \mu_1(t) * \rho_1(t) \\ r(t) * V_2(t) - \mu_2(t) * \rho_2(t) \\ \dots \\ r(t) * V_L(t) - \mu_L(t) * \rho_L(t) \end{bmatrix} \qquad \begin{array}{l} \sum_i P_i(t) = 1, \quad P_i(t) \geq 0, \quad \forall i \\ \phi(t) \in [0,1] \end{array}$$

The first term of the objective function is the sum of the average cost of setting up and maintaining $V_i(t)$ virtual circuits on every link $i$, $i = 1, ..., L$, at time $t$. The second term of the objective function is the average loss of not admitting new virtual circuits into the network at time $t$. The third term of the objective function is the sum of the average cost of having $N_i(t)$ packets on every link $i$, $i = 1, ..., L$, at time $t$. When this cost is integrated over a time period, it reflects the average cost of packet delay during that time period. Finally, the last term of the objective function is the negative profit of packet throughput $\mu_i(t) * \rho_i(t)$ on every link $i$, $i = 1, ..., L$, at time $t$. To proceed, we assume that $\rho_i(t)$ is defined for $N_i(t) \geq 0$, is concave, monotonically increasing and twice differentiable in $N_i$ with $\lim_{N_i \to \infty} \rho_i(N_i) = 1$.

Necessary conditions for optimality are provided by the Pontryagin maximum principle [30, 2]. The Hamiltonian function of the state $X$, the controls $P_i, \phi$ and the costate variables $Q_{V,i}, Q_{N,i}$ at time $t$ is

$$
\begin{aligned}
H(X,P,\phi,Q,t) &= \sum_i C_{V,i}(t)*V_i(t) - C_\phi(t)*\phi(t)*\gamma(t) \; + \\
&+ \sum_i C_{N,i}(t)*N_i(t) - \sum_i C_{\mu,i}(t)*\mu_i(t)*\rho_i(t) \; + \\
&+ \sum_i Q_{V,i}(t)*[\gamma(t)*\phi(t)*P_i(t) - \delta(t)*V_i(t)] \; + \\
&+ \sum_i Q_{N,i}(t)*[r(t)*V_i(t) - \mu_i(t)*\rho_i(t)]
\end{aligned}
$$

$$
\textit{such that} \; \sum_i P_i(t) = 1, \quad P_i(t) \geq 0, \quad \phi(t) \in [0,1] \qquad \forall i.
$$

also, the costate variables must satisfy the adjoint equations

$$
\dot{Q}_{V,i}(t) = -\frac{\partial H(X,P,\phi,Q,t)}{\partial V_i(t)} = -\{\, C_{V,i}(t) - Q_{V,i}(t)*\delta(t) + Q_{N,i}(t)*r(t) \,\}
$$

$$
\begin{aligned}
\dot{Q}_{N,i}(t) = -\frac{\partial H(X,P,\phi,Q,t)}{\partial N_i(t)} &= -\{\, C_{N,i}(t) - C_{\mu,i}(t)*\mu_i(t)*\frac{d\rho_i(t)}{dN_i(t)} - \\
&\quad -Q_{N,i}(t)*\mu_i(t)*\frac{d\rho_i(t)}{dN_i(t)} \,\}
\end{aligned}
$$

In this section, we formulated the integrated window flow controlled virtual circuit routing and congestion control, and the link error rate effect on the performance as an optimal control problem. In the next section we will find the optimum feedback control policies.

## 4. ADAPTIVE OPTIMAL CONTROL POLICIES

In this section we find the optimum virtual circuit routing, congestion control and window flow control policies. For given $N_i(t)\ \forall i$, the minimization of the Hamiltonian with respect to the routing probabilities is equivalent to the following minimization problem

$$
\begin{array}{ll}
\textit{minimize} & \sum_i Q_{V,i}(t)*P_i(t) \\
\textit{with respect to} & P_i(t) \quad \forall i \\
\textit{such that} & \sum_i P_i(t) = 1, \quad P_i(t) \geq 0 \quad \forall i
\end{array}
$$

A similar problem has also been considered by Filipiak [5, 6, 8], for routing packets in datagram networks and by Tipper & Sundareshan [26] for virtual circuit routing. However, the state of our system depends both on the expected number of virtual circuits and packets in the network and we also optimize a multiobjective function. Therefore, our costate variables are different than theirs. So, let the minimum costate variable be $Q_V^*(t) = \min_i\{Q_{V,i}(t)\}$, then the optimum routing probabilities will be :

$$
P_i^*(t) \begin{cases} > 0 & \text{only if } Q_{V,i}(t) = Q_V^*(t). \\ = 0 & \text{o.w.} \end{cases}
$$

Therefore link $i$ will be used only if it has the minimum costate variable $Q_{V,i}(t) = Q_V^*(t)$.
If only one link $i$ achieves the minimum costate variable $Q_V^*(t)$ , then $P_i^*(t) = 1$.
However, if more than one link achieves the minimum costate variable $Q_V^*(t)$,
then for these links $0 < P_i^*(t) < 1$ and for the rest links $P_j^*(t) = 0 \quad \forall j \neq i$.

Next, we solve the problem for the steady state ($T \to \infty$). The mean number of packets for link $i$ at steady state is $N_i = \rho_i/(1-\rho_i)$. Rewriting, we have the utilization of link $i$ at steady state $\rho_i = N_i/(1+N_i)$. At steady state, the costate variables must satisfy

$$\dot{Q}_{N,i} = 0 \Rightarrow \quad -\{C_{N,i} - C_{\mu,i} * \mu_i * \frac{d\rho_i}{dN_i} - Q_{N,i} * \mu_i * \frac{d\rho_i}{dN_i}\} = 0$$

$$Q_{N,i} = \frac{C_{N,i}}{\mu_i * \frac{d\rho_i}{dN_i}} - C_{\mu,i} = \frac{C_{N,i} * (1+N_i)^2}{\mu_i} - C_{\mu,i}$$

$$\dot{Q}_{V,i} = 0 \Rightarrow \quad -\{C_{V,i} - Q_{V,i} * \delta + Q_{N,i} * r\} = 0$$

$$Q_{V,i} = \frac{C_{V,i} + Q_{N,i} * r}{\delta} = \frac{C_{V,i} + C_{N,i} * (1+N_i)^2/\mu_i - C_{\mu,i}}{\delta}$$

**Routing Rule :** a new virtual circuit is routed through the link that has the minimum costate variable:

$$\frac{C_{V,i} + C_{N,i} * (1+N_i)^2/\mu_i - C_{\mu,i}}{\delta} = \min_j\{\frac{C_{V,j} + C_{N,j} * (1+N_j)^2/\mu_j - C_{\mu,j}}{\delta}\}$$

A simple example for routing to two links with $C_{V,1} = C_{V,2} = 0$, $C_{\mu,1} = C_{\mu,2} = 0$ and $\mu_2 = 4 * \mu_1$ is given in Fig. 5.

Now, we substitute these optimum routing probabilities $P_i^* \quad \forall i$ into the Hamiltonian and minimize the Hamiltonian with respect to the admission probability $\phi$

*minimize* $\quad [-C_\phi + \sum_i Q_{V,i} * P_i^*] * \phi$

*with respect to* $\quad \phi$

*such that* $\quad \phi \in [0,1]$.

When the routing probabilities achieve their optimum values $P_i^* \quad \forall i$, we have

$$\sum_i Q_{V,i} * P_i^* = Q_V^*$$

Then the minimization of the Hamiltonian with respect to the admission probability $\phi$ is equivalent to

*minimize* $\quad [-C_\phi + Q_V^*] * \phi$

*with respect to* $\quad \phi$

*under the constraints* $\quad \phi \in [0,1]$.

The optimum admission probability is

$$\phi^* = \begin{cases} 0 & \text{for } Q_V^* > C_\phi \\ 1 & \text{for } Q_V^* < C_\phi \\ 0 \le \phi \le 1 & \text{for } Q_V^* = C_\phi \end{cases}$$

**Admission Rule :** a new virtual circuit is admitted into the network only if

$$\min_i\{\frac{C_{V,i} + C_{N,i} * (1+N_i)^2/\mu_i - C_{\mu,i}}{\delta}\} < C_\phi$$

In this section, we have derived the optimum virtual circuit routing and congestion control policies that also depend on the window flow control parameters and the link error rates.

## 5. CONCLUSIONS

In this paper, we have presented *nonlinear non-stationary queueing models of virtual circuit networks*, by explicitly considering the interaction among the virtual circuit and packet processes. We *formulated the integrated virtual circuit routing, congestion, flow and error control* problem as an optimal control problem. We set up a *multiple objective function* and we solved it using the Pontryagin maximum principle. Finally, we derived *feedback optimal control policies* for virtual circuit network management.

We are currently working on simulating the proposed nonlinear non-stationary queueing models and optimal control policies, as well as extending them for virtual circuit networks with end to end window flow control. We are also studying the impact of the window flow control parameters and investigating under what conditions the assumption of independent exponential service times on the links is valid. We will use this queueing theory-based state space approach for the integrated virtual circuit network management and control.

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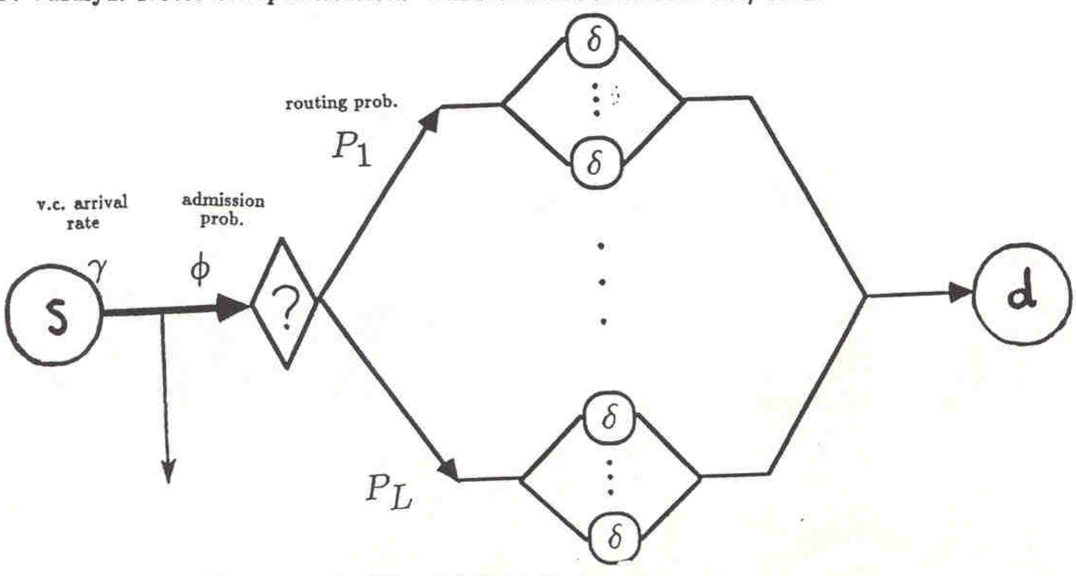



Fig. 1 Virtual circuit process.

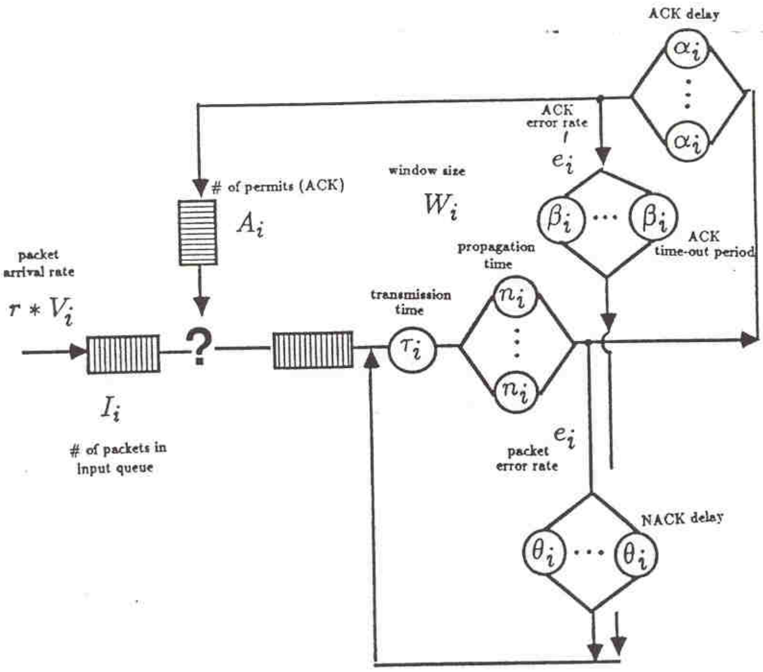



Fig. 2 Packet process for link $i$.

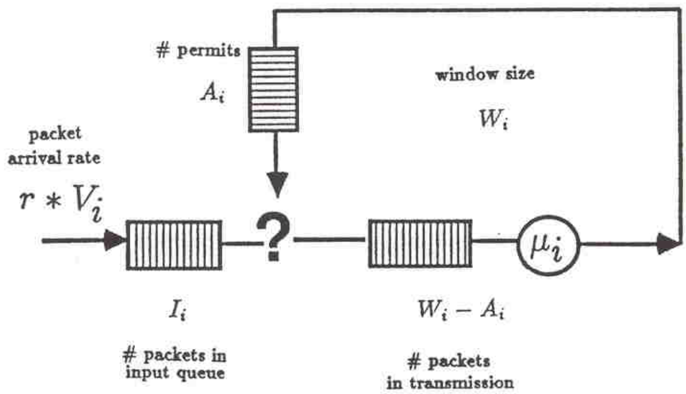
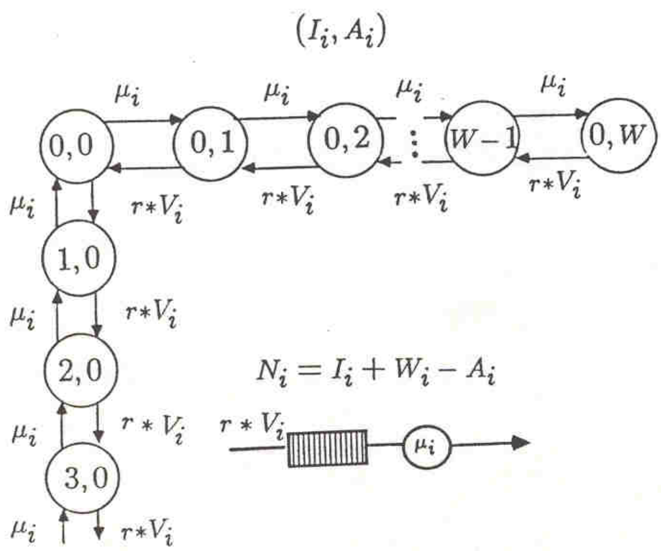



Fig. 3 Approximate model for link $i$.



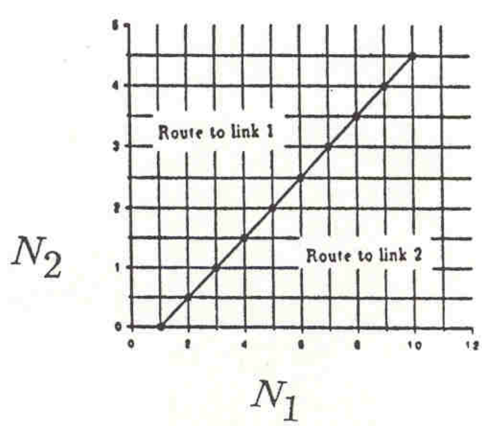




Fig. 4 $M/M/1$ model for link $i$.

Fig. 5 Threshold routing to two links.