

## Dynamic Hierarchical Management of ATM Networks

Anastasios A. Economides

University of Macedonia

Thessaloniki 54006, GREECE

Phone +3031-891799, E-Mail: economid@macedonia.uom.gr

**Abstract** - Asynchronous Transfer Mode (ATM) networks will transfer many different traffic classes with different service and performance requirements. There will be different users and network managers, each one with different control rights and power. For example, a single chief administrator may control the ATM network, but the users may try to exploit it for their satisfaction ignoring the smooth operation of the network. If the network management ignores this egoistic behavior of the users, many problems may arise in the network operation.

This paper provides the framework for formulating such scenarios using game theory, optimal control theory and queueing theory. The chief administrator knows the strategies of the users and acts first. Then the users choose their strategies trying to improve their own performance.

### I. INTRODUCTION

Previous studies on networks assume that there is a single main administrator who takes all decisions for the control and management of the network. However, in a network there are many different decision-makers, each one with different power, requirements and control. For example, even if there is a single network manager, there are still many different users and each one tries to exploit the network for his exclusive benefit. The network management fails if it ignores the existence of all these different decision-makers. In this paper, we consider this reality and define the network management as a hierarchical game between a leader and his followers. The leader minimizes his cost taking in mind the reactions of the followers to his decisions. He knows the strategy of his followers and acts first. Then, the followers act trying to minimize their costs on the limits imposed by the leader.

We have introduced a similar approach for quasi-static (not dynamic) network problems in [3,4]. A different approach for quasi-static network problems where the decision makers have equal power uses the Nash game theory [1,2,5,6,9,10,11,12,13,14]. In this paper, we formulate dynamic hierarchical network problems as dynamic Stackelberg games.

In ATM networks there will be various multimedia traffic that require transfer and even processing in a computer system. If some newly arriving traffic requires processing, then the decision-maker who controls this traffic must decide where he will process it. This is the

load sharing problem. However, he accepts new traffic into the network only if there are free available resources to satisfy the requirements of this new traffic and simultaneously continue to provide the agreed quality of service level for his traffic that is already in the network. The problem of controlling the newly incoming traffic is the admission control problem. For his traffic that is accepted into the network, the next problem is to determine the route that it will follow to its destination so that the desired requirements of quality of service are satisfied. This is the routing problem. Previous studies ignore the interaction of these three problems and examine each one stand alone or only the admission control and routing problem [7]. In this paper, we examine the joint dynamic admission control, routing and load sharing problem in ATM networks.

In section II, we provide the dynamic model for ATM networks and formulate the joint problem on the state space using queueing theory and optimal control theory. In section III, we formulate the hierarchical problem as a dynamic Stackelberg game and provide the conditions in 3 theorems for open-loop and closed-loop Stackelberg equilibrium. Finally, in section IV, we conclude on the proposed approach for dynamic hierarchical management of ATM networks.

### II. DYNAMIC MODEL OF ATM NETWORKS

We consider that the network state continuously changes due to the dynamic traffic fluctuations. We describe this dynamic traffic flow using nonlinear dynamic models on the state space. Other dynamic flow models have been also used for the routing problem with a single decision-maker [7,8]. We consider that the traffic is ATM cells of 53 Bytes (424 bits) and arrive at the network links and nodes with Poisson distribution.

Let traffic of decision-maker  $c$  that requires processing arrive at node  $[s.]$  with arrival rate  $\gamma_{[s.]}^c(t) \geq 0$  at instant  $t$ . Fraction  $\psi_{[s.d]}^c(t) \geq 0$  of this traffic is assigned for processing at computer system  $[.d]$ . Of course, the sum of these fractions to all candidate computer systems must be equal to 1: 
$$\sum_{[.d]} \psi_{[s.d]}^c(t) = 1.$$

So, the  $c$  traffic from node  $[s.]$  to computer system  $[.d]$  is  $\gamma_{[s.]}^c(t)\psi_{[s.d]}^c(t)$ . In addition to node  $[s.]$ , other nodes send traffic for processing to computer system  $[.d]$ . Thus, the total  $c$  traffic that arrives to computer system  $[.d]$  for processing is: 
$$\lambda_{[.d]}^c(t) = \sum_{[s.]} \gamma_{[s.]}^c(t)\psi_{[s.d]}^c(t).$$

The mean number of  $c$  cells at the computer system  $[.d]$ ,  $N_{[.d]}^c(t)$ , increases during a time period by the mean

number of  $c$  cells that arrive during this period, while it decreases by the mean number of  $c$  cells that depart (after being processed at rate  $C_{[d]}$  cells/sec) during this period. Consequently, we write the differential equation for the mean number of  $c$  cells that describes the state of the computer system  $[d]$  [14]:  $N_{[d]}^c(t) = \lambda_{[d]}^c(t) - C_{[d]}^c$ \*

$$* \frac{2N_{[d]}^c(t) \left( -\sum_k N_{[d]}^k(t) + \sqrt{1 + \left( \sum_k N_{[d]}^k(t) \right)^2} \right)}{1 - \sum_k N_{[d]}^k(t) + \sqrt{1 + \left( \sum_k N_{[d]}^k(t) \right)^2}} \quad (1)$$

At steady state the equation above satisfies the formula for the mean number of  $c$  cells for  $M/D/1$  queues with multiple classes.

In addition to the traffic that requires processing, there is traffic that requires transfer to some destination. In other words, decision-maker  $c$  controls traffic only for transfer and traffic for transfer plus processing. Hence, he must decide which route his total traffic should follow to the destinations. Furthermore, he may also decide to reject a fraction of his traffic. Therefore, for the total  $c$  traffic from node  $[s.]$  to destination  $[d]$ , he rejects a fraction  $\phi_{o[sd]}^c(t) \geq 0$  and sends a fraction  $\phi_{\pi[sd]}^c(t) \geq 0$  through route  $\pi[sd]$ . Obviously, the sum of these rejection and routing fractions must be equal to 1:  $\phi_{o[sd]}^c(t) + \sum_{\pi[sd]} \phi_{\pi[sd]}^c(t) = 1$ .

So, if the  $c$  traffic that has to be transferred from node  $[s.]$  to destination  $[d]$  is  $\gamma_{[sd]}^c(t) \geq 0$ , then the total  $c$  traffic from node  $[s.]$  to destination  $[d]$  is:  $\gamma_{[sd]}^c(t) + \gamma_{[s.]}^c(t)\psi_{[sd]}^c(t)$ . Then, the rejected  $c$  traffic is:  $\lambda_{o[sd]}^c(t) = (\gamma_{[sd]}^c(t) + \gamma_{[s.]}^c(t)\psi_{[sd]}^c(t))\phi_{o[sd]}^c(t)$ . Also, the  $c$  traffic routed through route  $\pi[sd]$  is:  $(\gamma_{[sd]}^c(t) + \gamma_{[s.]}^c(t)\psi_{[sd]}^c(t))\phi_{\pi[sd]}^c(t)$ .

As the traffic is routed through various routes, it traverses several network links and nodes. The traffic that passes through link  $ij$  is the sum of the traffic of all routes  $\pi[sd]$  between every source-destination  $[sd]$  that pass through this link:  $\lambda_{ij}^c(t) = \sum_{[sd]} \sum_{\pi[sd], ij \in \pi[sd]} (\gamma_{[sd]}^c(t) + \gamma_{[s.]}^c(t)\psi_{[sd]}^c(t))\phi_{\pi[sd]}^c(t)$ .

The mean number of  $c$  cells on link  $ij$ ,  $N_{ij}^c(t)$ , increases during a time period by the mean number of  $c$  cells that arrive during this period while it decreases by the mean number of  $c$  cells that depart (after being transmitted at rate  $C_{ij}$  cells/sec) during this period. Thus, we describe the dynamic state of link  $ij$  with the following differential equation for the mean number of  $c$  cells [14]:  $N_{ij}^c(t) = \lambda_{ij}^c(t) - C_{ij}^c$ \*

$$* \frac{2N_{ij}^c(t) \left( -\sum_k N_{ij}^k(t) + \sqrt{1 + \left( \sum_k N_{ij}^k(t) \right)^2} \right)}{1 - \sum_k N_{ij}^k(t) + \sqrt{1 + \left( \sum_k N_{ij}^k(t) \right)^2}} \quad (2)$$

Likewise, the traffic that traverses node  $i$  is the sum of all traffic of all routes  $\pi[sd]$  between all source-

destination pairs  $[sd]$  that pass through this node:  $\lambda_i^c(t) = \sum_{[sd]} \sum_{\pi[sd], i \in \pi[sd]} (\gamma_{[sd]}^c(t) + \gamma_{[s.]}^c(t)\psi_{[sd]}^c(t))\phi_{\pi[sd]}^c(t)$

Similarly, we describe the dynamic state of node  $i$  (which has speed  $C_i$  cells/sec) with the following differential equation for the mean number of  $c$  cells [14]:  $N_i^c(t) = \lambda_i^c(t) - C_i^c$ \*

$$* \frac{2N_i^c(t) \left( -\sum_k N_i^k(t) + \sqrt{1 + \left( \sum_k N_i^k(t) \right)^2} \right)}{1 - \sum_k N_i^k(t) + \sqrt{1 + \left( \sum_k N_i^k(t) \right)^2}} \quad (3)$$

Writing all these differential equations (1), (2), (3) for both decision-makers, the leader  $\alpha$  and the follower  $\beta$ , for all network resources, in a vector form, we describe the network state:  $\mathbf{N}(t) = \mathbf{f}(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$  where  $\mathbf{N} \geq 0$  is the vector of the mean number of cells at all network resources and  $(\mathbf{\Phi}, \mathbf{\Psi}) \in \mathbf{S}$  (or  $(\mathbf{\Phi}^\alpha, \mathbf{\Psi}^\alpha) \in \mathbf{S}^\alpha$  for the leader and  $(\mathbf{\Phi}^\beta, \mathbf{\Psi}^\beta) \in \mathbf{S}^\beta$  for the follower) is the vector of the control variables at the space where they are defined.

Having described the dynamic network state, we define the cost function for each decision-maker  $c$  from the initial time instant  $t_0$  up to the final time instant  $t_f$ :  $J^c(\mathbf{\Phi}, \mathbf{\Psi}) = \int_{t_0}^{t_f} g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) dt$ , where

$$\begin{aligned} & g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) \\ &= \sum_{ij} g_{ij}^c(t, \lambda_{ij}^c(t)) + \sum_i g_i^c(t, \lambda_i^c(t)) \\ & - \sum_{[sd]} g_{o[sd]}^c(t, (1 - \phi_{[sd]}^c(t))) - \sum_{[d]} g_{[d]}^c(t, \lambda_{[d]}^c(t)) \end{aligned}$$

Hence, the cost function for decision-maker  $c$  is composed from the delay cost at every link  $ij$  and every node  $i$ , minus the income from traffic admission between every source-destination  $[sd]$  and the income from traffic processing at computer systems  $[d]$ .

Let also define his Hamiltonian:

$$H^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) = g^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}) + \mathbf{P}^c \mathbf{f}(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi})$$

where  $\mathbf{P}^c = [\dots P_{[d]}^{c,k} \dots P_{ij}^{c,k} \dots P_i^{c,k} \dots]$  are the variables related to the network state equations.

Finally, his Lagrangian is:

$$\begin{aligned} L^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c, \mathbf{Q}^c) &= H^c(t, \mathbf{X}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) \\ &+ \sum_{[s.]} Q_{[s.]}^c \cdot \left[ 1 - \sum_{[d]} \psi_{[sd]}^c \right] \\ &+ \sum_{[sd]} Q_{[sd]}^c \cdot \left[ 1 - \phi_{o[sd]}^c - \sum_{\pi[sd]} \phi_{\pi[sd]}^c \right] \end{aligned}$$

with  $\phi_{o[sd]}^c, \phi_{\pi[sd]}^c, \psi_{[sd]}^c \geq 0 \quad \forall \pi[sd], [sd], c$ .

where  $Q^c = [\dots Q_{[sd]}^c \dots Q_{[s]}^c \dots]$  is the vector of the multipliers that are related with the constraints of the control variables for decision-maker  $c$ .

### III. HIERARCHICAL MANAGEMENT

Having a dynamic model that describes the state of the ATM network with a leader  $\alpha$  and a follower  $\beta$ , with fixed initial time  $t_0$  and final time  $t_f$ , we define the dynamic joint problem as an optimal control problem. Due to space limitation, we omit the proofs.

**Theorem 1:** *If for every decision-maker  $c$ , his Hamiltonian  $H^c(t, N, \Phi, \Psi, P(t))$  is differentiable and convex w.r.t.  $(N, \Phi^c, \Psi^c) \in (\mathbb{R}^n, S^c) \forall t \in [t_0, t_f]$ , for every constant value of the  $(\Phi^k, \Psi^k) \in S^k$ , then  $(\Phi^*(t), \Psi^*(t)) \in S$  is the Stackelberg equilibrium if and only if it solves the following Optimal Control Problem:*

$$\begin{aligned} & \text{minimize } \int_{t_0}^{t_f} g^\alpha(t, N(t), \Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t)) dt \\ & \text{w.r.t. } (\Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t)) \\ & \text{s.t. } \dot{N}(t) = f(t, N(t), \Phi(t), \Psi(t)) \quad N(t_0) = N_0 \\ & (\Phi^\alpha(t), \Psi^\alpha(t)) \in S^\alpha, \quad (\Phi^\beta(t), \Psi^\beta(t)) \in S^\beta \end{aligned}$$

$$\begin{aligned} & \int_{t_0}^{t_f} g^\beta(t, N(t), \Phi^\alpha(t), \Phi^\beta(t), \Psi^\alpha(t), \Psi^\beta(t)) dt = \\ & = \min_{(\tilde{\Phi}^\beta(t), \tilde{\Psi}^\beta(t)) \in S^\beta} \int_{t_0}^{t_f} g^\beta(t, N(t), \Phi^\alpha(t), \tilde{\Phi}^\beta(t), \Psi^\alpha(t), \tilde{\Psi}^\beta(t)) dt \end{aligned}$$

**Theorem 2:** *Let for each decision-maker  $c$ , his Hamiltonian  $H^c(t, N, \Phi, \Psi, P(t))$  is differentiable and convex w.r.t.  $(N, \Phi^c, \Psi^c) \in (\mathbb{R}^n, S^c) \forall t \in [t_0, t_f]$ , for every constant value of  $(\Phi^k, \Psi^k) \in S^k$ . If  $(\Phi^*(t, N_0), \Psi^*(t, N_0)) = (\Phi^*(t), \Psi^*(t))$  is an open-loop Stackelberg equilibrium and  $\{N^*(t), t \in [t_0, t_f]\}$  is the corresponding state trajectory, then  $\exists P^c(t) : [t_0, t_f] \rightarrow \mathbb{R}^n, \forall c$  continuous and continuously differentiable vector function, such that  $\forall t \in [t_0, t_f]$ :*

$$\begin{aligned} & \text{minimize } \int_{t_0}^{t_f} g^\alpha(t, N(t), \Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t)) dt \\ & \text{w.r.t. } (\Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t), Q^\beta(t)) \\ & \text{s.t. } \dot{N}(t) = f(t, N(t), \Phi(t), \Psi(t)), \quad N(t_0) = N_0 \end{aligned}$$

$$\begin{aligned} \dot{P}^\beta(t) &= -\nabla_N H^\beta(t, N, \Phi(t), \Psi(t), P^\beta(t)) \\ P^\beta(t_f) &= 0 \end{aligned}$$

$$\begin{aligned} & \left[ \frac{\partial H^\beta}{\partial \phi_{\sigma[sd]}^\beta} - Q_{[sd]}^\beta(t) \right] \cdot \phi_{\sigma[sd]}^\beta(t) = 0, \\ & \frac{\partial H^\beta}{\partial \phi_{\sigma[sd]}^\beta} - Q_{[sd]}^\beta(t) \geq 0 \quad \forall [sd] \end{aligned}$$

$$\begin{aligned} & \left[ \frac{\partial H^\beta}{\partial \phi_{\pi[sd]}^\beta} - Q_{[sd]}^\beta(t) \right] \cdot \phi_{\pi[sd]}^\beta(t) = 0, \\ & \frac{\partial H^\beta}{\partial \phi_{\pi[sd]}^\beta} - Q_{[sd]}^\beta(t) \geq 0 \quad \forall \pi[sd], [sd] \end{aligned}$$

$$\begin{aligned} & \left[ \frac{\partial H^\beta}{\partial \psi_{[sd]}^\beta} - Q_{[s]}^\beta(t) \right] \cdot \psi_{[sd]}^\beta(t) = 0, \\ & \frac{\partial H^\beta}{\partial \psi_{[sd]}^\beta} - Q_{[s]}^\beta(t) \geq 0 \quad \forall [d], [s] \end{aligned}$$

$$\begin{aligned} & \phi_{\sigma[sd]}^\alpha(t) + \sum_{\pi[sd]} \phi_{\pi[sd]}^\alpha(t) = 1, \\ & \phi_{\sigma[sd]}^\alpha(t), \phi_{\pi[sd]}^\alpha(t) \geq 0 \quad \forall \pi[sd], [sd] \end{aligned}$$

$$\sum_{[d]} \psi_{[sd]}^\alpha(t) = 1, \quad \psi_{[sd]}^\alpha(t) \geq 0 \quad \forall [d], [s]$$

$$\begin{aligned} & \phi_{\sigma[sd]}^\beta(t) + \sum_{\pi[sd]} \phi_{\pi[sd]}^\beta(t) = 1, \\ & \phi_{\sigma[sd]}^\beta(t), \phi_{\pi[sd]}^\beta(t) \geq 0 \quad \forall \pi[sd], [sd] \end{aligned}$$

$$\sum_{[d]} \psi_{[sd]}^\beta(t) = 1, \quad \psi_{[sd]}^\beta(t) \geq 0 \quad \forall [d], [s]$$

**Theorem 3:** *Let for each decision-maker  $c$ ,  $g^c(t, N, \Phi, \Psi), f(t, N, \Phi, \Psi)$ , are continuously differentiable w.r.t.  $(N, \Phi, \Psi) \in (\mathbb{R}^n, S) \forall t \in [t_0, t_f]$ . If  $(\Phi^*(t, N, N_0), \Psi^*(t, N, N_0)) = (\Phi^*(t), \Psi^*(t)) \in S$  is a closed-loop without memory Stackelberg equilibrium such that  $(\Phi^{c*}(t, N, N_0), \Psi^{c*}(t, N, N_0))$  is continuously differentiable with respect to  $N \in \mathbb{R}^n, \forall c, t \in [t_0, t_f]$  and  $\{N^*(t), t \in [t_0, t_f]\}$  is the corresponding state trajectory then  $\exists P^c(t) : [t_0, t_f] \rightarrow \mathbb{R}^n, \forall c$ , continuous and piecewise continuously differentiable vector function such that  $\forall t \in [t_0, t_f]$ :*

$$\text{minimize } \int_{t_0}^{t_f} g^\alpha(t, N(t), \Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t)) dt$$

$$\text{w.r.t. } (\Phi^\alpha(t), \Psi^\alpha(t), \Phi^\beta(t), \Psi^\beta(t), Q^\beta(t))$$

$$\text{s.t. } \dot{N}(t) = f(t, N(t), \Phi(t), \Psi(t)), \quad N(t_0) = N_0$$

$$\dot{P}^\beta(t) = -\nabla_N H^\beta(t, N, \Phi(t, N, N_0), \Psi(t, N, N_0), P^\beta(t))$$

$$P^\beta(t_f) = 0$$

$$\begin{aligned} & \left[ \frac{\partial H^\beta}{\partial \phi_{\sigma[sd]}^\beta} - Q_{[sd]}^\beta(t) \right] \cdot \phi_{\sigma[sd]}^\beta(t) = 0, \\ & \frac{\partial H^\beta}{\partial \phi_{\sigma[sd]}^\beta} - Q_{[sd]}^\beta(t) \geq 0 \quad \forall [sd] \end{aligned}$$

$$\left[ \frac{\partial H^\beta}{\partial \phi_{\pi[sd]}^\beta} - Q_{[sd]}^\beta(t) \right] \cdot \phi_{\pi[sd]}^\beta(t) = 0,$$

$$\frac{\partial H^\beta}{\partial \phi_{\pi[sd]}^\beta} - Q_{[sd]}^\beta(t) \geq 0 \quad \forall \pi[sd], [sd]$$

$$\left[ \frac{\partial H^\beta}{\partial \psi_{[sd]}^\beta} - Q_{[s.]}^\beta(t) \right] \cdot \psi_{[sd]}^\beta(t) = 0,$$

$$\frac{\partial H^\beta}{\partial \psi_{[sd]}^\beta} - Q_{[s.]}^\beta(t) \geq 0 \quad \forall [d], [s.]$$

$$\phi_{o[sd]}^\alpha(t) + \sum_{\pi[sd]} \phi_{\pi[sd]}^\alpha(t) = 1,$$

$$\phi_{o[sd]}^\alpha(t), \phi_{\pi[sd]}^\alpha(t) \geq 0 \quad \forall \pi[sd], [sd]$$

$$\sum_{[d]} \psi_{[sd]}^\alpha(t) = 1, \quad \psi_{[sd]}^\alpha(t) \geq 0 \quad \forall [d], [s.]$$

$$\phi_{o[sd]}^\beta(t) + \sum_{\pi[sd]} \phi_{\pi[sd]}^\beta(t) = 1,$$

$$\phi_{o[sd]}^\beta(t), \phi_{\pi[sd]}^\beta(t) \geq 0 \quad \forall \pi[sd], [sd]$$

$$\sum_{[d]} \psi_{[sd]}^\beta(t) = 1, \quad \psi_{[sd]}^\beta(t) \geq 0 \quad \forall [d], [s.]$$

#### IV. CONCLUSIONS

In this paper, we examine the dynamic joint admission control, routing and load sharing problem in ATM networks with two hierarchical decision-makers. The leader acts first minimizing his cost and knowing the strategy of the follower. Then the follower reacts trying to minimize his cost in the limits imposed by the leader. First, we introduce a state space model of nonlinear differential equations that describe the dynamic traffic flow in ATM networks. Then we formulate the hierarchical dynamic joint problem using game theory. Finally, we present the conditions for open and closed-loop Stackelberg equilibrium.

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