

The Economics of Network Industries

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

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First published 2001

Printed in the United States of America

Typeface Computer Modern 10/12 pt. *System* L^AT_EX 2_ε [AU]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data

Shy, Oz.

The economics of network industries / Oz Shy.

p. cm.

Includes bibliographical references and index.

ISBN 0-521-80095-1 – ISBN 0-521-80500-7 (pb)

1. Computer industry. 2. Computer networks. 3. Electronic data interchange. 4.
Electronic commerce. I. Title.

HD9696.8.A2 S55 2000

303.48'33—dc21

00-063044

ISBN 0 521 80095 1 hardback

ISBN 0 521 80500 7 paperback

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Chapter 1

Introduction to Network Economics

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1.1 Overview of Network Industries

This book is about markets. Not really a special type of market, since there are many markets for goods and services that satisfy the characteristics of what we call network products. These markets include the telephone, email, Internet, computer hardware, computer software, music players, music titles, video players, video movies, banking services, airline services, legal services, and many more. This book is also about social interaction and how it affects consumers’ choices of products and services they buy.

The main characteristics of these markets which distinguish them from the market for grain, dairy products, apples, and treasury bonds are:

- Complementarity, compatibility and standards.
- Consumption externalities.
- Switching costs and lock-in.
- Significant economies of scale in production.

Complementarity, compatibility and standards

Computers are not useful without having monitors attached, or without having software installed. CD players are not useful without CD titles, just as cameras are not useful without films. Stereo receivers are useless without speakers or headphones, and airline companies will not be able to sell tickets without joining a particular reservation system. All these examples demonstrate that, unlike bread which can be consumed without wine or other types of food, the markets we analyze in this book supply goods that must be consumed together with other products (software and hardware). In the literature of economics, such goods and services are called *complements*. Complementarity means that consumers in these markets are shopping for *systems* (e.g., computers and software, cameras and film, music players and cassettes) rather than individual products. The fact that consumers are buying systems composed of hardware and software or complementary components allows firms to devise all sorts of strategies regarding competition with other firms. A natural question is to ask, for example, is whether firms benefit from designing machines that can work with machines produced by rival firms.

On the technical side, the next question to ask would be how complements are produced? In order to produce complementary products they must be *compatible*. The CD album must have the same specification as CD players, or otherwise it can't be played. A parallel port at the back of each computer must generate the same output voltage as the voltage required for inputting data into a printer attached to this port. Trains must fit on the tracks, and software must be workable with a given operating system. This means that complementary products must operate on the same *standard*. This creates the problem of *coordination* as how firms agree on the standards. The very fact that coordination is needed has the potential of creating some antitrust problems. As in some cases firms may need to coordinate their decisions and while doing that they may find themselves engaging in price fixing.

Complementarity turns to be a crucial factor in the markets for information goods. For example, people who subscribe to the *Private-Pilot* magazine are likely to be interested in fashion clothing catalogs, just like people who read the *New York Times* are likely to be interested in real-estate and interior decoration magazines. Advertising agencies have understood these complementarities for quite some time, and make use of these complementarities to attract more customers. For example, the publishers of real-estate magazines could benefit from purchasing the list of names and addresses of the subscribers to the *New York Times*, and send them sample copies to attract their attention. These information

complementarities become more and more important with the increase in the use of the Internet for advertising and shopping purposes. For example, those who browse in commercial Internet sites offering toys for sale, such as www.etoys.com, are likely to be interested in browsing through Internet sites offering children clothing. Thus, the toy sites are likely to sell the list of their site visitors to children clothing stores.

Externalities

The reader should ask herself the following question: Would I subscribe to a telephone service knowing that nobody else subscribes to a telephone service? The answer should be: Of course not! What use will anyone have from having a telephone for which there is no one to talk to? Would people use e-mail knowing that nobody else does? Would people purchase fax machines knowing that nobody else has such a machine? These examples demonstrate that the utility derived from the consumption of these goods is affected by the number of other people using similar or compatible products. Note that this type of externalities is not found in the market for tomatoes, or the market for salt, as the consumption of these goods does not require compatibility with other consumers. Such externalities are sometimes referred to as *adoption or network externalities*.

The presence of these standard-adoption effects can profoundly affect market behavior of firms. The precise nature of the market outcome (e.g., consumers' adoption of a new standard) depends on how consumers form expectations on the size of the network of users. The reliance on joint-consumer expectations generates multiple equilibria where in one equilibrium all consumers adopt the new technology, whereas in the other no one adopts it. Both equilibria are "rational" from the consumers' viewpoint as they reflect the best response to the decisions made by all other consumers in the market. A good example for this behavior is the fax machine, which has been used in the 1950s by flight service stations to transmit weather maps every hour on the hour (transmission of single page took about one hour that time). However, fax machines remained a niche product until the mid-1980s. During a five-year period, the demand and supply of fax machines exploded. Before 1982 almost no one had a fax machine, but after 1987, the majority of businesses had one. The Internet exhibited the same pattern of adoption. The first e-mail message was sent in 1969, but adoption did not take off until the mid-1980s. The Internet did not take off until 1990, however, from 1990 Internet traffic more than doubles every year. All these examples raise a fundamental question, which is when to expect a new technology to catch on. A related question to ask is in the presence of adoption exter-

nalities, what should be the minimal number of users (the critical mass) needed for inducing all potential consumers to adopt the technology.

Switching costs and lock-in

Learning to master a particular operating system such as Windows, UNIX, DOS, or a Macintosh takes time (depending on the level of the user). It is an established fact that users are very much annoyed by having to switch between operating systems. To some consumers, switching operating systems is as hard as learning a new language. On the production side, producers heavily depend on the standards used in the production of other components of the system. For example, airline companies rely on spare parts and service provided by aircraft manufacturers. Switching costs are significant in service industries as well. Several estimates provided in this book show that the cost associated with switching between banks (i.e., closing an account in one bank, and opening an account and switching the activities to a different bank) could reach 6 percent of the average account balance. In all of these cases, we say that users are *locked-in*. Of course, lock-in is not an absolute term. The degree of lock-in is found by calculating the cost of switching to a different service or adopting a new technology, since these costs determine the degree in which users are locked in a given technology. We call these costs *switching costs*.

There are several types of switching costs that affect the degree of lock-in. Shapiro and Varian (1999) provide a nice classification of the various lock-ins.

Contracts: Users are sometimes locked into contracts for service, supplying parts, and buying spare parts. Switching costs amount to the damages and compensation that must be paid by the party who breaks the contract.

Training and learning: Consumers are trained to use products operating on a specific standard. Switching costs would include learning and training people, as well as lost productivity resulting from adopting a new system.

Data conversion: Each piece of software generates files that are saved using a particular digital format. Once a new software is introduced, a conversion software may be needed in order to be able to use it. Notice that the resulting switching cost increases over time as the collection of data may grow over time.

Search cost: One reason why people do not switch very often is that they would like to avoid the cost of searching and shopping for new products.

Loyalty cost: Switching technology may result in losing some benefits such as preferred customers' programs, for example, frequent-flyer mileage.

Switching costs affect price competition in two opposing ways. First, if consumers are already locked-in using a specific product, firms may raise prices knowing that consumers will not switch unless the price difference exceeds the switching cost to a competing brand. Second, if consumers are not locked in, brand-producing firms will compete intensively by offering discounts and free complimentary products and services in order to attract consumers who later on will be locked in the technology.

In the presence of switching costs, once the critical mass is achieved and the sales of the product take off, we say that the seller has accumulated an *installed base* of consumers, which is the number of consumers who are locked in the seller's technology. For example, AT&T's installed base is the number of customers subscribing to its long-distance service, where switching costs include the time and trouble associated with switching to, say, MCI's long-distance service.

Significant economies of scale

Software, or more generally any information has the highly noticeable production characteristic in which the production of the first copy involves a huge sunk cost (cost that cannot be recovered), whereas the second copy (third, fourth, and so on) costs almost nothing to reproduce. The cost of gathering the information for the Britannica encyclopedia involves more than one hundred years of research as well as the life-time work of a good number of authors. However, the cost of reproducing it on a set of CDs is less than five dollars. The cost of developing advanced software involves thousands of hours of programming time, however, the software can now be distributed without cost over the Internet. In economic terms, a very high fixed sunk cost, together with almost negligible marginal cost implies that the average cost function declines sharply with the number of copies sold out to consumers. This by itself means that a competitive equilibrium does not exist and that markets of this type will often be characterized by dominant leaders that capture most of the market.

Any student of intermediate microeconomics would clearly identify the major problem associated with modeling these markets, namely, that *these markets cannot function as competitive markets*, where by compet-

itive we take the usual interpretation of price-taking behavior. Therefore the purpose of this book is to develop simple theories that would explain the behavior of companies in these noncompetitive markets.

1.2 Welfare Aspects

1.2.1 Government intervention

From our discussion in Section 1.1 it is clear that competitive equilibria do not exist in markets for network products and services. This implies that the First-Welfare Theorem of classical economics cannot be applied. Moreover, even if a competitive equilibrium exists, the existence of consumption and production externalities would make this theorem inapplicable. Therefore, market failures may occur in these markets.

The distortions leading to these misallocation of resources could be generated by noncompetitive behavior of firms, or by the consumption externalities, for example where the industry standardizes on the Pareto-inferior standard. Despite these market imperfections, while reading this book the reader must bear in mind that the *existence of market failures does not imply that government intervention is needed*. In fact the following examples illustrate that government intervention may make things even worse. The FCC's attempt to impose the CBS color TV standard in 1950 has left 200 consumers with unusable TV sets after the market has rejected the government-chosen standard and switched to NBC's NTSC standard which is used until this very day. For about twenty years the Japanese Ministry of International Trade and Industry (MITI) poured millions of dollars into the research and development of a standard for a high-definition television (HDTV). Finally, in 1990 one station started broadcasting high-definition programs for a few hours every night using MITI's MUSE standard. The MUSE standard suffered from one major problem, namely, that it was an analog standard that has already been considered outdated in the early 1990s. Today, the Japanese are switching to a digital standard. Both of these examples highlight the fact that government intervention can be harmful.

From this discussion, it is clear why government intervention in standard setting is undesirable. Yes, it is true that market failures occur where an industry standardizes on a second-best technology. However, there is no guarantee that government intervention would guarantee a first-best standard selection. In fact, since politicians are financed partly by firms, governments may end up imposing Pareto inferior standards. Therefore, despite the market failures recognized in this book, the reader must bear in mind that the author of this book does *not* advocate government intervention in standard settings!

1.2.2 “Natural” monopolies versus access pricing

It has been argued during the 1950s until the early 1980s, both in the academic world and by policy and decision makers that industries like telephony, mail/post, cable TV, electricity, gas, and transportation are subjected to strong economies of scale production patterns (see for example Section 3.1), and should therefore be termed as *natural monopolies*. The strong academic support for such a view has led governments to license only one company in a given region, and in many cases for the entire country. Thus, until the early 1980s, most countries licensed a single company called *Public Telephone and Telegraph (PTT)* to provide telephony and mail deliveries. Cable TV in the U.S. and later in other countries followed the same pattern in which cable TV operators were given a geographical territory in which they were allowed to exercise a monopoly power. In order to avoid “excessive” monopoly charges, governments assigned regulating authorities and gave them a full power to determine prices based on production costs.

The major characteristic of natural monopolies is that these industries are subjected to strong economies of scale due to the significant investment in infrastructure needed to start the operation and a very small marginal cost for services produced over the existing infrastructure. More precisely, the idea behind natural monopolies is that it is a social waste to have each competing telephone company wiring its own network into each apartment building, where residents choose different carriers. Similarly, this argument held that it is socially undesirable to have more than one mail carrier reaching each neighborhood.

During the 1970s governments began realizing two major problems with the operations of these regulated service-providing (‘natural’) monopolies:

- (a) Service was relatively poor and was not improving at the pace of technological advance made in these industries. For example, consumers did not benefit from the introduction of fast hand-writing recognizing mail sorters in the sense that delivery time did not improve and stamp prices did not fall.
- (b) Regulators failed to control prices and other charges levied on consumers. Due to asymmetric information, the regulators failed to observe the true production cost faced by these service-providing firms, so these firms tended to inflate their reported production costs in order to lobby for high prices.

Thus, over the years governments began realizing that despite the significant economies of scale in production, competition may improve

social welfare, or at least consumer welfare, who stand to gain a lot from improved service and reduced prices.

The deregulation of the airline industry in 1979, the 1982 break up of the world largest telephone company, AT&T, in the United States, and the deregulation of these industries in Europe in the 1990s confirmed the view that the introduction of competition into these industries is welfare improving. Moreover, whereas sharp welfare improvement on the consumer side was expected from competition, regulator found out that competition hardly worsened anything on the production side. More precisely, the natural monopoly theory argues that a multi-firm industry is inefficient, since each firm will end up operating on the downward sloping part of its average cost curve due to less than optimal scale of production. However, this prediction on the inefficiency of production of a multi-firm industry turned out to be false. How come? Well, as it turned out the introduction of *access pricing* (see Section 5.3) preserved the efficient large-scale use of existing infrastructure by letting all firms use the existing infrastructure while paying access charges to the firm that owns and maintains the infrastructure.

Access pricing is now practiced in all network industries. MCI, SPRINT, and AT&T pay access charges to local phone companies in the United States for the termination of long-distance phone calls originated by their customers. Airline and railroad companies pay access fees for using airport gates and railroad tracks owned and maintained by competing firms. Norwegian electric-power producers are able to sell electricity to German users by accessing the German infrastructure in order to deliver electricity to German homes and factories. All these examples demonstrate that the introduction of competition did not leave existing and newly constructed infrastructure underutilized. In fact, it turned out that the introduction of competition together with the regulators' demand that the existing infrastructure will be available for use by all competitors for "reasonable" access charges led to even more efficient utilization of infrastructure by having different companies providing substitute or complementary services. All this leads us to conclude that letting industries be controlled by the so-called "natural monopolies" was inefficient. In fact the name itself, natural monopoly, is problematic since a monopoly is one form of market structure that is maintained by government intervention or the persistence of patent rights. Clearly, there is nothing "natural" in the formation of monopolies. Therefore this term is likely to disappear from the language used by regulators and professional and academic economists.

1.3 References and the Scientific Literature

My experience with the writing of my first book (Shy, 1996) has taught me that textbooks must be written by the author, since any attempt to merely “copy” papers published in the scientific journals and use them as chapters in a book just yields messy chapters. The reason for this lies in the fact that scientific papers are written mainly for the scientific community and are, therefore, written in a “different language” that does not fit into textbooks (or anything else).

For this reason, I took the task of simplifying the literature by building completely new models that are not based on calculus (derivatives, integrals, etc.). As the reader will find out, this task is not easy, since discrete price-competition models with heterogeneous consumers generally do *not* have a Nash-Bertrand equilibrium (see Proposition C.1 on page 308 for example). This is perhaps the major reason why economics journals are flooded with calculus-based models utilizing tedious algebra.

Because of that, I also limited the references to this literature made at the end of each chapter only to those which had some influence on the precise method of presentation that suits the potential readers of this book. Thus, the choice of which papers to cite and which not to cite *does not reflect the degree of importance* of the papers. Therefore, I beg the forgiveness of all those large number of researchers whose works are not cited, and ask them to understand that the sole goal of this book is to bring the economics of networks to a wider audience, which includes undergraduate students as well as researchers and graduate students who have a limited technical ability. However, I do wish to refer the interested reader to a large number of survey articles listed at the end of this chapter, and the Internet site <http://raven.stern.nyu.edu/networks>, which provides a complete list of related literature.

Finally, this is perhaps the right place to criticize the scientific literature (including my own) for the language it uses in published papers. After writing two books in the field of Industrial Organization it is clear to me that economists unnecessarily write models using tedious derivations with an unnecessarily large amount of algebra. The prevailing *myth* in the academic economics profession is that *complicated algebra implies that the argument made is robust*. Obviously, this widespread myth is rather silly and reflects the hypocrisy of our profession. I have two arguments against this prevailing myth: First, there is no such a thing called a robust model. Every model has its assumptions, which limit the applicability of the model. Second, more importantly, I claim that models that rely more on logic and less on algebra are more robust (more general in plain English) than models utilizing long equations with long derivatives exceeding in size the width of the paper they are printed on.

Thus, the arguments made in this book are simple, but they are not less general than the models published in the scientific literature.

1.4 Notation

Notation is classified into two groups: *parameters*, which are numbers that are treated as exogenous by the agents described in the model, and *variables*, which are endogenously determined. Thus, the purpose of every theoretical model is to define an equilibrium concept that yields a unique solution for these variables for given values of the model's parameters.

For example, production costs and consumers' valuations of products are typically described by parameters (constants), which are estimated in the market by econometricians and are taken exogenously by the theoretical economist. In contrast, quantity produced and quantity consumed are classical examples of variables that are endogenously determined meaning that they are solved within the model itself.

We now set the rule for assigning notation to parameters and variables. *Parameters are denoted by Greek letters, whereas variables are denoted by English letters.*

After setting this rule, we adhere to the famous statement that *all rules are meant to be broken* and state a few exceptions. For example, π will denote a firm's profit level, despite the fact that profit levels are variables that are endogenously-solved for within a specified model. After breaking all rules, Table 1.1 on page 12 provides some indication of the notation used throughout this book.

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Parameters		
Notation	Greek	Interpretation
λ	lambda	number of firms in an industry
ϕ	phi	fixed or sunk production cost
μ	mu	unit production cost
ψ	psi	productivity parameter
ρ	rho	revenue per customer
η	eta	a given population size
β	beta	basic utility derived from a product
α	alpha	intensity of the network externality
ω	omega	a consumer's income (wage)
τ	tau	a particular time period (e.g., $t = \tau$)
ϵ	epsilon	probability or a small number
δ	delta	differentiation (or switching) cost
Variables		
t		time period (e.g., $t = 1, 2, \dots$)
U		utility level of a single consumer
e		a consumer's expenditure
π_i	pi	profit level of firm i
p_i/f_i		price/fee charged by firm i
q_i		quantity produced by a firm i
Q		aggregate industry output
W		social welfare
Symbols		
$=$		equals by derivation
$\stackrel{\text{def}}{=}$		equals by definition
\approx		approximately equal
\neg		not (negation)
\implies		implies that
\iff		if and only if
Δ	Delta	a change in a variable/parameter
∂		partial derivative
\longrightarrow		approaches (converges) to
\in		is an element of the set
■		end-of-proof (<i>QED</i>)

Table 1.1: General notation for parameters, variables, and symbols.